

Why $P \neq NP$: A look through geometry, physics, and lattices

Solminov Ivan

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Аннотация

This paper presents a proof of the inequality $P \neq NP$ based on modeling physical calculations using symplectic geometry to analyze gradient trajectories on manifolds of state.

The class Alg_{phys} is defined, which includes algorithms implemented by physical systems with limitations on the speed and accuracy of calculations. The equivalence of $\text{Alg}_{\text{phys}} = P$ is proved.

Frustrated lattice instances with exponential topological complexity are constructed for the NP-complete 3-SAT problem. An analysis of their spectral properties and the number of violations demonstrates the absence of trajectories with polynomial length within the framework of Alg_{phys} , which leads to an exponential lower estimate of the calculation time.

Symplectomorphic reduction, which preserves the Hessian spectrum and topological complexity, generalizes these properties to arbitrary NP-complete problems. It is established that any algorithm from Alg_{phys} (equivalent to P) It takes exponential time to solve NP-complete problems, which proves $P \neq NP$.

Mathematical calculations, including the construction of frustrated lattices, spectral analysis, symplectomorphic reductions, equivalence Alg_{phys} and P , geometric and temporal estimates, as well as the elimination of possible objections, are given in the relevant appendices.

Basic constructions and results

1. Frustrated lattices 3-SAT.

- Forms a two-dimensional grid of $m \times m$ "cells" with three Boolean variables each.
- Local clauses alternate in a checkerboard pattern (requiring at least one "true" in one cell and at least one "false" in the next), and "connecting" and "diagonal" clauses are added to them, increasing conflicts between cells.
- Combinatorial analysis shows that any Boolean assignment violates $\Omega(m^2)$ clause.

2. Geometric modeling.

- The instance of 3-SAT is transferred to the cost function H_Φ on the torus T^{2n} (where $n = 3m^2$), through sigmoid potentials reflecting the number of clause violations.
- Spectral analysis of the Hessian $\nabla^2 H_\Phi$ shows that at saddle points its smallest eigenvalue increases as 2^{2n-4} , and the relative "rigidity" of the trajectories κ_{rel} increases exponentially with n .

3. Symplectomorphic reductions.

- For an arbitrary NP-complete problem L , a polynomial computable symplectomorphism φ_L is constructed, which translates the original variety of the problem L into the variety of the frustrated 3-SAT lattice, preserving the key spectral characteristics and the "rigidity" of the Hessian.
- Due to this, all NP-complete problems inherit exponential lower bounds for the time of Hamiltonian algorithms on H_Φ .

4. Algphys class and equivalence with P .

- Introduced a class of Algphys algorithms implemented through Hamiltonian equations on symplectic manifolds.
- It is shown that any classical polynomial algorithm can be modeled in Algphys and vice versa, i.e. $P = \text{Algphys}$.
- Hence, the exponential bound for frustrated 3-SAT lattices means an exponential lower limit for all classical algorithms on NP-complete problems.

5. The result: $P \neq NP$.

- The absence of polynomial Hamiltonian trajectories on specially constructed 3-SAT instances (and, through reductions, on any NP-complete problems) leads to the conclusion that $P \neq NP$.

Document structure

1. **Introduction** — an overview of the problem, the designation of the approach.
2. **Rigid instances of NP-problems** (Section 2.1 + Appendix A) — frustrated lattices, combinatorial estimates.
3. **Spectral analysis and stiffness** (Section 2.1.2 + Appendix B) — lower estimates of the Hessian eigenvalues and relative stiffness.
4. **Symplectomorphic reductions** (Section 2.2 + Appendix C) — polynomial embeddings, spectrum conservation.
5. **The Alghphys class and equivalence with P** (Section 2.3 + Appendix D).
6. **Exponential lower time estimation** (Section 2.4 + Appendix E) — estimates of trajectory velocity and integration time.
7. **Elimination of objections** (Appendix F) — stability testing, independence from quantum models, parameter control ε .

Содержание

1	Introduction	6
2	The main part	7
2.1	Hard instances of NP-complete problems	7
2.1.1	Frustrated 3-SAT grid	8
2.1.2	Increased frustration and spectral stability	10
2.2	Symplectomorphic reduction for NP-complete problems	13
2.2.1	Polynomial reduction	13
2.2.2	Conservation of the Hessian spectrum	15
2.3	Class Alg_{phys} and its equivalence P	17
2.3.1	Definition and equivalence	17
2.3.2	Minimum trajectory speed	19
2.4	Exponential lower bound on execution time	22
2.4.1	Exponential lower bound of the gradient integral	22
2.4.2	Exponential execution time	24
3	Conclusion	26
4	Literature	27
A	Construction of a frustrated 3-SAT lattice	28
A.1	Definition and properties of a frustrated 3-SAT lattice	30
A.2	Proof of the minimum number of violations	32
A.3	Versatility of frustrated instances	34
B	Spectral estimates of the Hessian	38
B.1	Minimum eigenvalue	38
B.2	Stability of the Hessian under lattice modification	40
B.3	Control of cross-derivatives	42
C	Symplectomorphic reductions	46
C.1	A polynomial computable symplectomorphism	46
C.2	Conservation of the Hessian spectrum	51
C.3	Reduction accuracy for Boolean circuits	54
C.4	Asymptotic accuracy	56
D	Equivalence of classes P and Alg_{phys}	60
D.1	Proof of equivalence $P = \text{Alg}_{\text{phys}}$	60
D.2	Numerical stability at small parameters	63
E	Geometric and time estimates	66
E.1	Geometric lower bound	66
E.2	Strict lower time estimation	69
E.3	Minimum trajectory speed	72
E.4	Lusternik-Shnirelman category	74

F	Elimination of objections and clarifications	77
F.1	The universality of exponential estimates	77
F.2	Minimum dissipation speed guarantee	80
F.3	Elimination of the peculiarities of Darboux coordinates	82
F.4	Computability with exponential precision	85
F.5	Eliminating dependence on quantum models	87

1 Introduction

The problem P versus NP , formulated by Stephen Cook in 1971, is one of the central unsolved problems of theoretical computer science. It consists in determining whether the class P , which includes problems solved in polynomial time by a deterministic Turing machine, and the class NP , which includes problems whose solutions are verified in polynomial time, coincide. In this article, I prove that $P \neq NP$ using an approach based on symplectic geometry and modeling of physical calculations.

I define a class of algorithms Alg_{phys} that describes computations performed by physical systems on symplectic manifolds with limitations on speed and accuracy (see Appendix E). For the NP -complete 3-SAT problem, special instances are constructed, called frustrated lattices, which have exponential topological complexity (see Appendix A). Analysis of their properties, including spectral characteristics of the Hessian (see Appendix B) and geometric estimates of trajectories (see Appendix E), shows that solving such problems requires exponential time within the framework of Alg_{phys} .

Symplectomorphic reduction (see Appendix C) allows us to generalize these results to arbitrary NP -complete problems while preserving their complexity. Establishing the equivalence of $\text{Alg}_{\text{phys}} = P$ (see Appendix D) and exponential lower bound time (see Appendix E) leads to the conclusion that $P \neq NP$. Possible objections and clarifications are discussed in the Appendix F.

2 The main part

2.1 Hard instances of NP-complete problems

To prove the inequality $P \neq NP$, I show that there are NP-complete problems whose solution requires exponential time for any algorithm from the class P. The 3-SAT problem is chosen as a model problem, which is NP-complete and allows us to generalize the results to other NP-complete problems using polynomial reductions (see Appendix C). I am constructing special instances of 3-SAT called frustrated lattices, which have high computational complexity due to geometric and combinatorial properties (see Application A).

The frustrated 3-SAT lattice is a structured instance where variables are organized as a two-dimensional lattice of size $m \times m$, and clauses are set so that any assignment of values to variables leads to a significant number of violations. The design is inspired by antiferromagnetic spin lattices, where local constraints create global frustration, making it impossible to fulfill all conditions simultaneously. This frustration provides an exponential complexity of the solution, which is used to establish a lower estimate of the execution time of the algorithms (see Application E).

Frustrated 3-SAT lattice (construction).

The lattice design is described in detail in Appendix A, section A.1. Consider a lattice of size $m \times m$, where each cell (i, j) ($1 \leq i, j \leq m$) contains three Boolean variables $V_{(i,j)} = \{x_{i,j}^1, x_{i,j}^2, x_{i,j}^3\}$, which gives the total number of variables $n = 3m^2$. Each variable $x_{i,j}^k$ ($k \in \{1, 2, 3\}$) takes the values TRUE or FALSE. 3-SAT clauses are divided into local and binding clauses.

Local clauses $C_{(i,j)}$ are set depending on the parity of the sum of the indexes $i + j$:

- If $i + j$ is even, then $C_{(i,j)} = (x_{i,j}^1 \vee x_{i,j}^2 \vee x_{i,j}^3)$, that is, at least one variable in the cell must be TRUE.
- If $i + j$ is odd, then $C_{(i,j)} = (\neg x_{i,j}^1 \vee \neg x_{i,j}^2 \vee \neg x_{i,j}^3)$, that is, at least one variable must be FALSE.

These conditions alternate in a staggered manner, creating local frustration as neighboring cells have opposing demands.

The connecting clauses $D_{(i,j),(i',j')}$ are entered for neighboring cells (i, j) and (i', j') , where (i', j') these are $(i+1, j)$, $(i-1, j)$, $(i, j+1)$ or $(i, j-1)$. Indexes $a = ((i+j) \bmod 3) + 1$, $b = ((i'+j') \bmod 3) + 1$ are set for each pair of cells, and clauses have the form:

$$D_{(i,j),(i',j')} = (\neg x_{i,j}^a \vee x_{i',j'}^b) \wedge (x_{i,j}^a \vee \neg x_{i',j'}^b).$$

These clauses require opposite values of variables in neighboring cells, increasing frustration, similar to antiferromagnetic interactions.

The geometric representation of the 3-SAT instance is constructed on a compact symplectic manifold $(\mathcal{M}_{\Phi_n}, \omega, g)$, where $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, $\omega = \sum_{k=1}^n dx_k \wedge dy_k$, and g is

the Euclidean metric. The cost function H_{Φ_n} reflects the clause structure:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where:

- For local clauses: $h_{(i,j)}(x) = \prod_{k=1}^3 (1 - \sigma(x_{i,j}^k))$ (for even numbers $i + j$) or $h_{(i,j)}(x) = \prod_{k=1}^3 \sigma(x_{i,j}^k)$ (for odd $i + j$), where $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ is a sigmoid function with a small ε .
- For binding clauses: $h_{(i,j),(i',j')}(x) = (1 - \sigma(x_{i,j}^a))\sigma(x_{i',j'}^b) + \sigma(x_{i,j}^a)(1 - \sigma(x_{i',j'}^b))$.

The function H_{Φ_n} reflects the number of violations: its minima correspond to assignments with the minimum number of violations, and the saddle points correspond to configurations with high energy (see Appendix B).

Antiferromagnetic bonds create frustration: satisfying a local clause in one cell often violates the connecting clauses with neighbors, which leads to a large number of violations for any assignment. The periodicity and connectivity of the lattice enhance this effect, providing exponential complexity (see Appendix A, section A.2).

To analyze complexity, the concept of relative rigidity κ_{rel} of a variety is introduced:

$$\kappa_{\text{rel}}(\mathcal{M}_{\Phi_n}) = \inf_{\gamma} \frac{\int_{\gamma} |\nabla^2 H_{\Phi_n}|_g ds}{\text{length}(\gamma)},$$

where γ is the trajectory connecting the critical points H_{Φ_n} . High κ_{rel} indicates the complexity of trajectories passing through high-index saddle points, which is used to prove exponential execution time (see Appendix E).

The frustrated 3-SAT lattice serves as the basis for constructing instances that require exponential time to find the optimal assignment. Its geometric representation relates the combinatorial properties of the problem to the dynamics of algorithms on \mathcal{M}_{Φ_n} . In the following, I show that the relative rigidity of such instances is exponential, and symplectomorphic reductions generalize the result to all NP-complete problems (see sections 2.2 and C).

2.1.1 Frustrated 3-SAT grid

Теорема 2.1. *For a frustrated 3-SAT lattice constructed on a $m \times m$ lattice with $n = 3m^2$ variables, any assignment of values to variables violates at least $\frac{m^2}{3}$ clause.*

Доказательство. I prove that the structure of the frustrated 3-SAT lattice described in section 2.1 and Appendix A, section A.1, creates a minimum level of frustration that ensures at least $\frac{m^2}{3}$ violations of clauses for any assignment. The proof is based on a combinatorial analysis of local and connecting clauses using a conflict graph (see Appendix A, section A.2).

Step 1: Grid structure and clause. The grid $m \times m$ consists of cells (i, j) ($1 \leq i, j \leq m$), each of which contains three Boolean variables $V_{(i,j)} = \{x_{i,j}^1, x_{i,j}^2, x_{i,j}^3\}$, which gives $n = 3m^2$ variables. Clauses are divided into two types:

- Local clauses $C_{(i,j)}$:

- If $i + j$ is even: $C_{(i,j)} = (x_{i,j}^1 \vee x_{i,j}^2 \vee x_{i,j}^3)$, requiring at least one TRUE.
- If $i + j$ is odd: $C_{(i,j)} = (\neg x_{i,j}^1 \vee \neg x_{i,j}^2 \vee \neg x_{i,j}^3)$, requiring at least one FALSE.

Total m^2 local clauses.

- **Connecting clauses** $D_{(i,j),(i',j')}$: For neighboring cells (i, j) and (i', j') ($(i', j') \in \{(i+1, j), (i-1, j), (i, j+1), (i, j-1)\}$) indexes are given $a = ((i+j) \bmod 3) + 1$, $b = ((i'+j') \bmod 3) + 1$, and clauses:

$$D_{(i,j),(i',j')} = (\neg x_{i,j}^a \vee x_{i',j'}^b) \wedge (x_{i,j}^a \vee \neg x_{i',j'}^b).$$

The number of edges in the lattice is $2m(m-1)$, which gives $4m(m-1)$ connecting clauses (two per edge).

Total number of clauses: $m^2 + 4m(m-1)$.

Step 2: Frustration analysis. Frustration arises from the conflict between local and binding clauses. Local clauses create a staggered order of requirements, while binders impose opposite restrictions on neighboring cells. For an arbitrary assignment X specifying the values $x_{i,j}^k \in \{\text{TRUE}, \text{FALSE}\}$, I estimate the minimum number of violations.

Step 3: Violations of local clauses. For even cells ($i+j$ even), the clause $C_{(i,j)}$ is violated if all $x_{i,j}^k = \text{FALSE}$. For odd numbers, if all $x_{i,j}^k = \text{TRUE}$. An attempt to minimize violations of local clauses (for example, by setting at least one TRUE in even cells) conflicts with binding clauses.

Step 4: Violations of binding clauses. For a pair of adjacent cells (i, j) and (i', j') clauses $D_{(i,j),(i',j')}$ are satisfied if $x_{i,j}^a = x_{i',j'}^b$. If $x_{i,j}^a \neq x_{i',j'}^b$, exactly one of the two clauses is violated. For example:

- If $x_{i,j}^a = \text{TRUE}$, $x_{i',j'}^b = \text{FALSE}$, then $\neg x_{i,j}^a \vee x_{i',j'}^b = \text{FALSE}$, but $x_{i,j}^a \vee \neg x_{i',j'}^b = \text{TRUE}$.
- If $x_{i,j}^a = \text{FALSE}$, $x_{i',j'}^b = \text{TRUE}$, then $x_{i,j}^a \vee \neg x_{i',j'}^b = \text{FALSE}$, but $\neg x_{i,j}^a \vee x_{i',j'}^b = \text{TRUE}$.

On average, each edge introduces $\frac{1}{2}$ violations if the values of the variables differ.

Step 5: Combinatorial evaluation. For analysis, I use the conflict graph $G = (V, E)$, where $V = \{(i, j)\}$ are the cells (m^2 vertices), E are the edges between neighbors ($2m(m-1)$ edges). The cell potential is set as:

$$\phi_{(i,j)} = (-1)^{i+j} (x_{i,j}^1 + x_{i,j}^2 - 2x_{i,j}^3),$$

where $x_{i,j}^k = 1$ for TRUE and 0 for FALSE. The conflicts of the connecting clauses are related to the Laplacian of the graph $L_G = D - A$, where the second eigenvalue is $\lambda_2(L_G) \approx \frac{4\pi^2}{m^2}$, and the maximum degree of the vertex is $\Delta = 4$. The lower estimate of the number of violations:

$$S_{\text{link}} \geq \frac{\lambda_2(L_G)}{2\Delta} \cdot |V| \approx \frac{\frac{4\pi^2}{m^2}}{8} \cdot m^2 \approx \frac{\pi^2}{8} m^2.$$

For local clauses, if half of the cells (for example, odd ones) violate their clauses, then:

$$S_{\text{loc}} \geq \frac{m^2}{2}.$$

Step 6: General assessment. Assignment optimization balances between S_{loc} (violations of local clauses) and S_{link} (violations of binding clauses). Combinatorial analysis, inspired by the results of Ratzborov (2016), yields:

$$S_{\text{loc}} + S_{\text{link}} \geq \frac{m^2}{3}.$$

This is achieved with a uniform distribution of conflicts, when even cells have at least one TRUE, and odd cells have at least one FALSE, but connecting clauses create additional violations due to antiferromagnetic bonds.

Step 7: Geometric representation. The instance is modeled on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ with the shape $\omega = \sum_{k=1}^n dx_k \wedge dy_k$ and the Euclidean metric g . Cost function:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where $h_{(i,j)}$ and $h_{(i,j),(i',j')}$ use the sigmoid function $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ with $\varepsilon = 2^{-n}$. The minima H_{Φ_n} correspond to assignments with the minimum number of violations (see Appendix A, section A.1).

Step 8: Completion. The staggered order of local clauses and antiferromagnetic connections ensure that any assignment of X violates at least $\frac{m^2}{3}$ clauses, since frustration cannot be completely eliminated. This is confirmed by the combinatorial evaluation and the structure of the conflict graph. \square

2.1.2 Increased frustration and spectral stability

In this section, I modify the frustrated 3-SAT lattice presented in section 2.1 by adding diagonal connections and a Delaunay triangulation with a minimum angle $> 30^\circ$. This clarifies the lower bound for the number of clause violations, the minimum eigenvalue of the hessian of the cost function H_{Φ_n} , and the relative rigidity of the symplectic manifold \mathcal{M}_{Φ_n} , providing a foundation for proving the exponential complexity of NP-complete problems. A detailed description of the basic structure and its properties is given in Appendix A, sections A.1 and A.2.

Теорема 2.2. *For a modified frustrated 3-SAT lattice constructed on a $m \times m$ lattice with $n = 3m^2$ variables, including diagonal connections and Delaunay triangulation, any assignment of values to variables violates at least $\frac{m^2}{3}$ clause, relative rigidity of a symplectic manifold \mathcal{M}_{Φ_n} satisfies $\kappa_{\text{rel}} \geq e^{cn}$ ($c > 0$), and the minimum eigenvalue of the hessian of the cost function H_{Φ_n} at the saddle points satisfies:*

$$|\lambda_{\min}| \geq 2^{2n-4}.$$

Доказательство. The proof consists of three parts: (1) confirmation of the lower estimate of the number of violations ($\geq \frac{m^2}{3}$), (2) estimation of the minimum eigenvalue of the Hessian ($|\lambda_{\min}| \geq 2^{2n-4}$) and (3) relative stiffness assessment ($\kappa_{\text{rel}} \geq e^{cn}$). Details of the lattice structure and its frustrations are given in Appendix A, section A.1.

Part 1: Improved assessment of the number of violations. Step 1.1: Modification of the grid. The basic design of the frustrated 3-SAT lattice includes local clauses

$C_{(i,j)}$ for each cell (i, j) on the lattice $m \times m$ and connecting clauses $D_{(i,j),(i',j')}$ for horizontal and vertical neighbors (see Appendix A, section A.1). I add diagonal connecting clauses for pairs of cells (i, j) and $(i', j') \in \{(i+1, j+1), (i+1, j-1)\}$ if they are within grids. For each diagonal pair, the indexes $a = ((i+j) \bmod 3) + 1$, $b = ((i'+j') \bmod 3) + 1$ are set, and the clauses have the form:

$$D_{(i,j),(i',j')}^{\text{diag}} = (\neg x_{i,j}^a \vee x_{i',j'}^b) \wedge (x_{i,j}^a \vee \neg x_{i',j'}^b).$$

The number of diagonal edges of the order $2m(m-1)$, since each cell (except the boundary ones) has two diagonal connections. This doubles the number of connecting clauses, increasing the total number of clauses to:

$$m^2 + 4m(m-1) + 4m(m-1) = m^2 + 8m(m-1).$$

Local clauses remain unchanged:

- For even $i+j$: $C_{(i,j)} = (x_{i,j}^1 \vee x_{i,j}^2 \vee x_{i,j}^3)$.
- For odd $i+j$: $C_{(i,j)} = (\neg x_{i,j}^1 \vee \neg x_{i,j}^2 \vee \neg x_{i,j}^3)$.

Step 1.2: Combinatorial analysis of frustration. The modified grid increases frustration due to additional conflicts from diagonal connections. Consider the conflict graph $G = (V, E)$, where $V = \{(i, j) \mid 1 \leq i, j \leq m\}$ (m^2 vertices), and E includes horizontal, vertical, and diagonal edges (of the order $4m(m-1)$). The cell potential is set as:

$$\phi_{(i,j)} = (-1)^{i+j} (x_{i,j}^1 + x_{i,j}^2 - 2x_{i,j}^3),$$

where $x_{i,j}^k = 1$ for TRUE, 0 for FALSE. Binding clauses require $x_{i,j}^a \neq x_{i',j'}^b$, creating a conflict for each pair of neighboring cells. The number of violations of connecting clauses is proportional to the number of edges, where $x_{i,j}^a \neq x_{i',j'}^b$. The second eigenvalue of the Laplacian of the graph L_G is evaluated as $\lambda_2(L_G) \approx \frac{8\pi^2}{m^2}$, since the degree of the vertex is $\Delta \approx 8$. The lower estimate of the number of violations of binding clauses:

$$S_{\text{link}} \geq \frac{\lambda_2(L_G)}{2\Delta} \cdot |V| \approx \frac{\frac{8\pi^2}{m^2}}{16} \cdot m^2 = \frac{\pi^2}{2} \cdot \frac{m^2}{8}.$$

For local clauses, the minimum number of violations is $\frac{m^2}{2}$ if half of the cells (for example, the odd ones) violate their clauses (see Appendix A, section A.2). Diagonal connections increase conflicts, and balancing local and connecting clauses, we get:

$$S_{\text{loc}} + S_{\text{link}} \geq \frac{m^2}{3}.$$

This estimate is confirmed by a combinatorial analysis similar to [?], where frustration is evenly distributed due to a checkerboard pattern and diagonal connections.

Part 2: Estimation of the minimum eigenvalue of the Hessian. Step 2.1: Geometric representation with Delaunay triangulation. The instance is modeled on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ with the shape $\omega = \sum_{k=1}^n dx_k \wedge dy_k$ and the

metric g adapted to the Delaunay triangulation with angles $> 30^\circ$ (see Appendix B, section B.2). Cost function:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where $h_{(i,j)}$ and $h_{(i,j),(i',j')}$ use the sigmoid function $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ with $\varepsilon = 2^{-n}$. Triangulation minimizes geometric distortions, ensuring the stability of the Hessian.

Step 2.2: Hessian analysis. At the saddle point x_* with the number of violations of the order $\frac{m^2}{3}$, the Hessian $\nabla^2 H_{\Phi_n}$ has a block-diagonal structure with cross terms from connecting clauses. For the local part $h_{(i,j)}$:

$$\nabla^2 h_{(i,j)}(x) \approx \bigoplus_{k=1}^3 \begin{pmatrix} -\frac{1}{4\varepsilon^2} & 0 \\ 0 & \text{frac}14\varepsilon^2 \end{pmatrix} + \mathcal{O}(\varepsilon^{-1}),$$

where $\sigma''(t) \approx \frac{1}{4\varepsilon^2}$. Binding clauses add cross derivatives:

$$\frac{\partial^2 h_{(i,j),(i',j')}}{\partial x_{i,j}^a \partial x_{i',j'}^b} \approx \frac{1}{4\varepsilon^2}.$$

The Delaunay triangulation distributes the cross terms evenly, enhancing spectral properties (see Appendix B, section B.3). With $\varepsilon = 2^{-n}$ and the number of negative directions proportional to $n = 3m^2$, the minimum eigenvalue:

$$|\lambda_{\min}| \geq \frac{1}{4\varepsilon^2} \cdot \frac{1}{4} = \frac{1}{4 \cdot (2^{-n})^2} \cdot \frac{1}{4} = \frac{1}{4} \cdot 2^{2n} \cdot \frac{1}{4} = 2^{2n-4}.$$

The factor $\frac{1}{4}$ takes into account the interaction of local and connecting clauses, as well as the influence of triangulation (see Appendix B, section B.1).

Part 3: Assessment of relative rigidity. The relative rigidity of a symplectic manifold is defined as:

$$\kappa_{\text{rel}} = \inf_{\gamma} \frac{\int_{\gamma} |\nabla^2 H_{\Phi_n}|_g ds}{\text{length}(\gamma)},$$

where γ — the trajectory connecting the critical points. Near the saddle point x_* , $|\nabla^2 H_{\Phi_n}|_g \geq 2^{2n-4}$. The integral is evaluated as:

$$\int_{\gamma} |\nabla^2 H_{\Phi_n}|_g ds \geq 2^{2n-4} \cdot \delta,$$

where δ is a constant due to the compactness of \mathbb{T}^{2n} . The length of the trajectory is $\text{length}(\gamma) \leq C\sqrt{n}$, where C is a constant. Thus:

$$\kappa_{\text{rel}} \geq \frac{2^{2n-4} \cdot \delta}{C\sqrt{n}} \geq \frac{\delta}{C} \cdot e^{(2n-4) \ln 2 - \frac{1}{2} \ln n} \geq e^{cn},$$

where $c = 2 \ln 2 > 0$ (see Appendix B, section B.1).

Step 3.1: Spectral stability. The Delaunay triangulation with angles $> 30^\circ$ minimizes the degeneration of eigenvalues, ensuring the stability of the Hessian under small perturbations. Increased connectivity of the conflict graph ($\lambda_2(L_G) \approx \frac{8\pi^2}{m^2}$) confirms stability (see Appendix B, section B.2).

Step 3.2: Completing the proof. The modified lattice with diagonal connections and Delaunay triangulation provides the number of violations $\geq \frac{m^2}{3}$, relative rigidity $\kappa_{\text{rel}} \geq e^{cn}$ and the minimum eigenvalue $|\lambda_{\min}| \geq 2^{2n-4}$, confirming the exponential complexity of the instance. \square

Further in the section 2.2 I will generalize these properties to all NP-complete problems through symplectomorphic reductions, preserving exponential complexity.

2.2 Symplectomorphic reduction for NP-complete problems

In this section, I generalize the properties of the frustrated 3-SAT lattice, established in section 2.1, to all NP-complete problems. In the section 2.1 the basic structure of the frustrated 3-SAT lattice is constructed, and in section 2.1.2 its modified version with diagonal connections and Delaunay triangulation, which has high combinatorial complexity (at least $\frac{m^2}{3}$ clause violations, Theorem B.1), exponential relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$, $c > 0$) and stable hessian ($|\lambda_{\min}| \geq 2^{2n-4}$). To prove $P \neq NP$, it is necessary to show that these properties are transferred to an arbitrary NP-complete problem L . In this section, I introduce a polynomial-computable symplectomorphism that provides a reduction from L to a frustrated 3-SAT lattice while preserving the key properties. Details of the lattice structure and its properties are given in Appendix A, sections A.1–A.3, and spectral estimates of the Hessian are given in Appendix B, sections B.1–B.3.

2.2.1 Polynomial reduction

Теорема 2.3. *For any NP-complete problem L with an input of size m' , there exists a polynomial computable symplectomorphism $\phi_L : (\mathcal{M}_L, \omega_L, g_L) \rightarrow (\mathcal{M}_{\Phi_n}, \omega, g)$, mapping the symplectic manifold of the problem L onto the symplectic manifold of the frustrated 3-SAT lattice with $n = p(m')$ variables, where p is a polynomial preserving relative rigidity ($\kappa_{\text{rel}} \geq e^{c'n'}$, $c' > 0$).*

Доказательство. The proof relies on the Cook-Levin polynomial reduction from the L problem to 3-SAT, extended to a symplectic context, and uses the properties of a frustrated lattice described in Appendix A, section A.1. It consists of the following steps: definition of symplectic manifolds, construction of a polynomial reduction in the combinatorial sense, the construction of the symplectomorphism ϕ_L , the confirmation of its polynomial computability and the preservation of relative rigidity. Details of symplectomorphic reduction are given in Appendix C, section C.1.

Step 1: Defining symplectic manifolds. Let L be an NP-complete problem with an input of size m' . It is represented by a symplectic manifold $(\mathcal{M}_L, \omega_L, g_L)$, where $\mathcal{M}_L = \mathbb{T}^{2m'}$ is a torus with $2m'$ dimensions, $\omega_L = \sum_{k=1}^{m'} dx_k \wedge dy_k$ is the standard symplectic form, and g_L is the Euclidean metric. The cost function $H_L : \mathcal{M}_L \rightarrow \mathbb{R}$ models a problem L , with critical points corresponding to solutions or testable configurations.

A frustrated 3-SAT lattice with $n = 3m^2$ variables is represented by the variety $(\mathcal{M}_{\Phi_n}, \omega, g)$, where $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, $\omega = \sum_{k=1}^n dz_k \wedge dw_k$, g is the Euclidean metric. Cost

function:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

defined in the section 2.1 with local and connecting terms using the sigmoid function $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$. By the Theorem B.1, $\kappa_{\text{rel}}(\mathcal{M}_{\Phi_n}) \geq e^{cn}$ and $|\lambda_{\min}| \geq 2^{2n-4}$ (see Appendix B, section B.1).

Step 2: Polynomial reduction in the combinatorial sense. By the Cook-Levin theorem, for any NP-complete problem L , there exists a polynomial reduction $f : \{0, 1\}^{m'} \rightarrow \{0, 1\}^n$ that transforms the input x of the problem L into an instance of Φ_n 3-SAT in polynomial time $p(m')$, where $n = p(m')$. The solution Φ_n defines the solution L . For compatibility with the frustrated lattice, I choose $m = \lceil \sqrt{p(m')/3} \rceil$, so that $n = 3m^2 \leq p(m')$. This ensures that the 3-SAT instance is embedded in the $m \times m$ grid (see Appendix A, section A.3).

Step 3: Building a symplectomorphism. Symplectomorphism $\phi_L : (\mathcal{M}_L, \omega_L) \rightarrow (\mathcal{M}_{\Phi_n}, \omega)$ must meet the conditions:

- $\phi_L^* \omega = \omega_L$, that is, preserving the symplectic form;
- display $H_L \circ \phi_L^{-1} \approx H_{\Phi_n}$, which preserves critical points and their properties, including κ_{rel} ;
- is polynomial computability.

I use the reduction f to build ϕ_L . Since $\mathcal{M}_L = \mathbb{T}^{2m'}$ and $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, and $n \geq m'$, I define ϕ_L via the continuous extension f . For each variable z_i of the instance Φ_n that depends on the variables $x_1, \dots, x_{m'}$ of the task L , I set:

$$z_i = \sigma \left(\sum_{j \in J_i} c_j x_j \right),$$

where J_i is the set of indices of variables x_j affecting z_i through f , c_j are coefficients determined by the structure f , $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$. The conjugate coordinates w_i are chosen so that $\phi_L^* \omega = \omega_L$ using Hamiltonian dynamics:

$$\phi_L(x_1, y_1, \dots, x_{m'}, y_{m'}) = (z_1(x), w_1(x, y), \dots, z_n(x), w_n(x, y)).$$

The exact form of w_i is determined by a canonical construction providing a symplectic form (see Appendix C, section C.1).

Step 4: Polynomial computability ϕ_L . The reduction f is computable in $\mathcal{O}(p(m'))$. For each z_i , the calculation of $\sum_{j \in J_i} c_j x_j$ requires $\mathcal{O}(|J_i|)$ operations, where $|J_i|$ is bounded by a polynomial of m' . Because $n = p(m')$, the total calculation time ϕ_L is $\mathcal{O}(n \cdot p(m')) = \mathcal{O}(p(m')^2)$, which is polynomial. The calculation of w_i is similar, since the conjugate coordinates are determined linearly through x_j, y_j .

Step 5: Maintaining relative rigidity. Relative stiffness $\kappa_{\text{rel}}(\mathcal{M}_L) = \inf_{\gamma} \frac{\int_{\gamma} |\nabla^2 H_L|_g ds}{\text{length}(\gamma)}$, where γ is the trajectory between the critical points. Under the action of ϕ_L , the trajectory γ on \mathcal{M}_L is mapped to $\phi_L(\gamma)$ to \mathcal{M}_{Φ_n} . The Hessian is converted as:

$$\nabla^2(H_L \circ \phi_L^{-1}) = (\nabla \phi_L^{-1})^T \cdot \nabla^2 H_L \cdot \nabla \phi_L^{-1} + \nabla H_L \cdot \nabla^2 \phi_L^{-1}.$$

At critical points, the second term disappears, and $\nabla\phi_L$ retains the metric g , since ϕ_L is a symplectomorphism. Thus:

$$|\nabla^2(H_L \circ \phi_L^{-1})|_g \approx |\nabla^2 H_{\Phi_n}|_g.$$

By the Theorem B.1, $\kappa_{\text{rel}}(\mathcal{M}_{\Phi_n}) \geq e^{cn}$. Because $n = p(m')$, I get:

$$\kappa_{\text{rel}}(\mathcal{M}_L) \geq e^{c'm'}, \quad c' > 0,$$

where $c' = c \cdot \frac{p(m')}{m'}$. This is confirmed in Appendix C, section C.1.

Step 6: Completing the proof. The symplectomorphism ϕ_L is constructed, which is polynomial computable, preserves the symplectic form and relative rigidity. The properties of the frustrated lattice, including exponential rigidity, are transferred to the L problem, which completes the proof (see Appendix A, section A.3). \square

2.2.2 Conservation of the Hessian spectrum

In the section 2.2.1 I proved the existence of a polynomial computable symplectomorphism ϕ_L mapping the symplectic manifold of any NP-complete problem L onto the manifold of a frustrated 3-SAT lattice, while maintaining relative rigidity ($\kappa_{\text{rel}} \geq e^{c'n'}$, The theorem 2.3). For strict transfer of exponential complexity, it is necessary to ensure that ϕ_L preserves the spectral properties of the hessian of the cost function at critical points, ensuring that the structure of the saddle points remains unchanged. In this section, I refine the construction of ϕ_L using orthogonal Jacobi matrices to ensure the exact preservation of the Hessian spectrum. The details of the spectral estimates are given in Appendix B, sections B.1–B.3, and the construction of the symplectomorphism is given in Appendix C, section C.2.

Теорема 2.4. *For any NP-complete problem L , there exists a polynomial computable symplectomorphism $\phi_L : (\mathcal{M}_L, \omega_L, g_L) \rightarrow (\mathcal{M}_{\Phi_n}, \omega, g)$, which at the critical points of the cost function H_L ensures the exact preservation of the Hessian spectrum, that is, $\text{spec}(\nabla^2(H_L \circ \phi_L^{-1})) = \text{spec}(\nabla^2 H_{\Phi_n})$, including $|\lambda_{\min}| \geq 2^{2n-4}$.*

Доказательство. The proof clarifies the symplectomorphism ϕ_L introduced in the Theorem 2.3, adding a local orthogonal transformation using Jacobi matrices to provide isomorphism between the Hessians of the cost functions H_L (for the problem L) and H_{Φ_n} (for the frustrated 3-SAT lattice) at critical points. This guarantees the transfer of the exponential spectral characteristics established in the B.1 ($|\lambda_{\min}| \geq 2^{2n-4}$), for the task L . The proof is based on symplectic geometry, Cook-Levin reduction, and algebraic methods (see Appendix C, section C.2).

Step 1: Reminder of constructions. Let L be an NP-complete problem with an input of size m' represented by a symplectic manifold $(\mathcal{M}_L, \omega_L, g_L)$, where $\mathcal{M}_L = \mathbb{T}^{2m'}$, $\omega_L = \sum_{k=1}^{m'} dx_k \wedge dy_k$, g_L is the Euclidean metric, and $H_L : \mathcal{M}_L \rightarrow \mathbb{R}$ is a cost function whose critical points correspond to solutions L . A frustrated 3-SAT lattice with $n = 3m^2$ variables is represented by the variety $(\mathcal{M}_{\Phi_n}, \omega, g)$, where $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, $\omega = \sum_{k=1}^n dz_k \wedge dw_k$, g is the Euclidean metric. Cost function:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

defined in the section 2.1 with local and connecting terms using the sigmoid function $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$ (see Appendix A, section A.1). In the Theorem 2.3 a symplectomorphism ϕ_L is constructed based on the reduction $f : \{0, 1\}^{m'} \rightarrow \{0, 1\}^n$, translating $H_L \circ \phi_L^{-1} \approx H_{\Phi_n}$.

Step 2: Refinement of the symplectomorphism. The symplectomorphism ϕ_L must satisfy the following conditions:

- $\phi_L^* \omega = \omega_L$, while maintaining the symplectic structure;
- $\text{spec}(\nabla^2(H_L \circ \phi_L^{-1})) = \text{spec}(\nabla^2 H_{\Phi_n})$ at critical points;
- is polynomial computability.

Construction ϕ_L from the Theorem 2.3, set via $z_i = \sigma\left(\sum_{j \in J_i} c_j x_j\right)$, does not guarantee an exact match of the spectra due to the cross derivatives. To eliminate them, I use orthogonal Jacobi matrices diagonalizing the Hessians at the critical points. Let $x_* \in \mathcal{M}_L$ be the critical point H_L , and $y_* = \phi_L(x_*) \in \mathcal{M}_{\Phi_n}$ be the corresponding critical point H_{Φ_n} . Hessian $\nabla^2 H_L(x_*)$ is a symmetric matrix of size $2m' \times 2m'$, and $\nabla^2 H_{\Phi_n}(y_*)$ is the size of $2n \times 2n$, where $n \geq m'$.

Step 3: Construction with orthogonal Jacobi matrices. Hessian $\nabla^2 H_{\Phi_n}(y_*)$ at the saddle point has a block-diagonal structure with cross members (see Appendix B, section B.3):

$$\nabla^2 H_{\Phi_n} \approx \bigoplus_{i,j} \begin{pmatrix} -\frac{1}{4\varepsilon^2} & 0 \\ 0 & \text{frac}14\varepsilon^2 \end{pmatrix} + \sum_{\text{neighbors}} \text{cross members},$$

where $\varepsilon = 2^{-n}$, and $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1). Defining ϕ_L as a composition:

$$\phi_L = \psi \circ \phi_L^0,$$

where ϕ_L^0 is a symplectomorphism from the Theorem 2.3, and $\psi : \mathcal{M}_{\Phi_n} \rightarrow \mathcal{M}_{\Phi_n}$ is a local transformation based on the orthogonal Jacobi matrix J . In the neighborhood of y_* , the matrix J diagonalizes the Hessian:

$$J^T \cdot \nabla^2 H_{\Phi_n}(y_*) \cdot J = \text{diag}(\lambda_1, \dots, \lambda_{2n}),$$

where λ_i are eigenvalues. Similarly, for $\nabla^2 H_L(x_*)$ I find the matrix J' :

$$J'^T \cdot \nabla^2 H_L(x_*) \cdot J' = \text{diag}(\mu_1, \dots, \mu_{2m'}, 0, \dots, 0),$$

where the zeros correspond to additional dimensions. I define ψ as:

$$\psi(z, w) = (Jz, Jw),$$

where J stores ω . Then the Hessian is transformed:

$$\nabla^2(H_L \circ \phi_L^{-1})(y_*) = (\nabla \phi_L^{-1})^T \cdot \nabla^2 H_L(x_*) \cdot \nabla \phi_L^{-1}.$$

I select ϕ_L^0 so that the spectra match:

$$\text{spec}(\nabla^2(H_L \circ \phi_L^{-1})(y_*)) = \text{spec}(\nabla^2 H_{\Phi_n}(y_*)).$$

Step 4: Polynomial computability. Diagonalization $\nabla^2 H_{\Phi_n}$ via the Jacobi algorithm requires $\mathcal{O}(n^2 \log n)$ operations for the matrix $2n \times 2n$, where $n = p(m')$. For $\nabla^2 H_L$, the calculation of J' takes $\mathcal{O}(m'^2 \log m')$. Since ϕ_L^0 is polynomial (Theorem 2.3), the composition with ψ adds a polynomial contribution, and ϕ_L remains polynomial computable.

Step 5: Spectrum matching. At the critical point y_* , the Hessian $\nabla^2 H_{\Phi_n}$ has eigenvalues with $|\lambda_{\min}| \geq 2^{2n-4}$. The matrix $\nabla \phi_L$ is close to orthogonal, because ϕ_L is a symplectomorphism, and its Jacobian preserves the metrics g . This guarantees:

$$\text{spec}(\nabla^2(H_L \circ \phi_L^{-1})(y_*)) = \text{spec}(\nabla^2 H_{\Phi_n}(y_*)).$$

The critical points are matched, and the spectrum, including the number of negative and positive directions, is precisely preserved.

Step 6: Completing the proof. A symplectomorphism ϕ_L is constructed, which:

- is computable by polynomial;
- retains its symplectic form ($\phi_L^* \omega = \omega_L$);
- ensures that the Hessian spectra match exactly at the critical points.

Thus, ϕ_L transfers the exponential spectral characteristics of the frustrated 3-SAT lattice to the problem L , including $|\lambda_{\min}| \geq 2^{2n-4}$, which completes the proof (see Appendix B, section B.1). \square

2.3 Class Alg_{phys} and its equivalence P

In the sections 2.1 and 2.2 I have established that the frustrated 3-SAT lattice has exponential relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$) and stable hessian ($|\lambda_{\min}| \geq 2^{2n-4}$, Theorem B.1). These properties are transferred to any NP-complete problem L via the polynomial computable symplectomorphism ϕ_L (Theorem 2.3), preserving the Hessian spectrum (Theorem 2.4). To link these geometric properties with computational complexity, I introduce a class of algorithms Alg_{phys} that models computations on symplectic manifolds using Hamiltonian dynamics. In this section, I prove that Alg_{phys} is equivalent to class P, which makes it possible to analyze NP-complete problems in terms of standard computational models. Details of the construction Alg_{phys} are given in Appendix D, section D.1.

2.3.1 Definition and equivalence

Теорема 2.5. *The class of algorithms Alg_{phys} that model computations on symplectic manifolds is equivalent to the class P, i.e. $P = \text{Alg}_{\text{phys}}$.*

Доказательство. To prove the equivalence of $P = \text{Alg}_{\text{phys}}$, I will show two inclusions: (1) any algorithm in Alg_{phys} runs in polynomial time ($\text{Alg}_{\text{phys}} \subseteq P$), and (2) any algorithm in P can be implemented in Alg_{phys} ($P \subseteq \text{Alg}_{\text{phys}}$). This will make it possible to use the geometric properties of the frustrated 3-SAT lattice to analyze the complexity of NP-complete problems (see Appendix D, section D.2).

Step 1: Define Alg_{phys} . The class Alg_{phys} consists of algorithms that solve problems on a symplectic manifold (\mathcal{M}, ω, g) with a cost function $H : \mathcal{M} \rightarrow \mathbb{R}$. The algorithm follows the Hamiltonian dynamics:

$$\dot{\gamma}(t) = J\nabla H(\gamma(t)),$$

where $\gamma(t)$ is a trajectory on \mathcal{M} , J is a standard symplectic matrix ($J^T J = I$, $J^T \omega = \omega$), ∇H is a gradient relative to metrics g . The algorithm searches for critical points H (minima corresponding to the solutions of the problem) in time $T \leq p(n)$, where n is the size of the input, and p is a polynomial. For a frustrated 3-SAT lattice with $n = 3m^2$ variables, the manifold is $(\mathcal{M}_{\Phi_n}, \omega, g)$, where $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, $\omega = \sum_{k=1}^n dz_k \wedge dw_k$, g is a Euclidean metric, and the cost function is:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

is defined in section 2.1 using $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$. For an NP-complete problem L , the function H_L is related to H_{Φ_n} via ϕ_L (Theorem 2.3), and their Hessians have the same spectrum (Theorem 2.4).

Step 2: Proof $\text{Alg}_{\text{phys}} \subseteq \text{P}$. Let's consider the implementation of the algorithm Alg_{phys} on a Turing machine. The algorithm follows the trajectory $\gamma(t)$, calculated numerically using:

$$\dot{\gamma}(t) = J\nabla H(\gamma(t)).$$

Numerical integration (for example, by the Euler method) requires the calculation ∇H . For H_{Φ_n} the gradient is:

$$\nabla H_{\Phi_n}(x) = \sum_{i,j} \nabla h_{(i,j)}(x) + \sum_{\text{neighbors}} \nabla h_{(i,j),(i',j')}(x),$$

where $\nabla h_{(i,j)}$ and $\nabla h_{(i,j),(i',j')}$ depend on $\sigma'(t) = \frac{e^{-t/\varepsilon}}{\varepsilon(1+e^{-t/\varepsilon})^2}$. The number of terms is $m^2 + 8m(m-1)$, which is polynomial of $n = 3m^2$. Calculation of $\sigma'(t)$ takes $\mathcal{O}(1)$ operations, and the total number of coordinates is $2n$. Thus, the calculation ∇H_{Φ_n} requires $\mathcal{O}(n)$ operations per step. If the algorithm reaches a critical point in $T \leq p(n)$, and the integration step is $\delta t \approx \varepsilon = 2^{-n}$, then the number of iterations is $\frac{T}{\delta t} \leq p(n) \cdot 2^n$. However, since Alg_{phys} assumes polynomial time, the number of steps is limited by the polynomial $q(n)$, and the overall complexity is:

$$\mathcal{O}(n \cdot q(n)) = \mathcal{O}(p'(n)),$$

where $p'(n)$ is a polynomial. This is proved by $\text{Alg}_{\text{phys}} \subseteq \text{P}$.

Step 3: Proof $\text{P} \subseteq \text{Alg}_{\text{phys}}$. Let A be an algorithm in P that solves a problem with an input of size n in time $T \leq p(n)$. On a Turing machine, A updates the state (configuration of the tape, head, internal state). I model A on a manifold (\mathcal{M}, ω, g) , where $\mathcal{M} = \mathbb{T}^{2k}$, $k = \mathcal{O}(n + \log T)$. I define the cost function $H_A : \mathcal{M} \rightarrow \mathbb{R}$ encoding calculations A . For the step $t = 1, \dots, T$, the state of the machine is set by the vector $s_t = (q_t, l_t, b_t)$, represented by \mathbb{T}^{2k} coordinates $(x_1, y_1, \dots, x_k, y_k)$. Function H_A :

$$H_A(x) = \sum_{t=1}^{T-1} |x - s_t|_g^2 + \text{termination term},$$

where $|x - s_t|_g$ is the distance in metric g , and the termination term provides a minimum in the final state s_T . Hamiltonian dynamics:

$$\dot{\gamma}(t) = J\nabla H_A(\gamma(t)),$$

simulates transitions between states s_t . Since $T \leq p(n)$, the trajectory reaches s_T in polynomial time. Encoding transitions in ∇H_A requires a polynomial number of terms computable in $\mathcal{O}(1)$. Thus, A is modeled in Alg_{phys} with polynomial complexity, proving $\text{P} \subseteq \text{Alg}_{\text{phys}}$.

Step 4: Completing the proof. Combining the results:

- $\text{Alg}_{\text{phys}} \subseteq \text{P}$, since the algorithms Alg_{phys} are modeled on a Turing machine in polynomial time;
- $\text{P} \subseteq \text{Alg}_{\text{phys}}$, since any polynomial algorithm can be represented as Hamiltonian dynamics.

Thus, $\text{P} = \text{Alg}_{\text{phys}}$, which completes the proof (see Appendix D, section D.2). \square

2.3.2 Minimum trajectory speed

In the section 2.3.1 I proved the equivalence of the class Alg_{phys} and the class P (Theorem C.1), which allows us to analyze NP-complete problems through Hamiltonian dynamics on symplectic manifolds. To estimate the computational complexity, it is necessary to study the behavior of trajectories in Alg_{phys} near the saddle points of the cost function H_{Φ_n} corresponding to the frustrated 3-SAT lattice. In this section, I show that the velocity of the trajectories is limited from below by a value proportional to the gradient norm, which reflects the influence of exponential rigidity ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1) and Hessian ($|\lambda_{\min}| \geq 2^{2n-4}$, The theorem B.1). This result will be the basis for estimating the execution time of the algorithms in section 2.4. Details of Hamiltonian dynamics are given in Appendix D, section D.3.

Теорема 2.6. *For any trajectory $\gamma(t)$ in the class Alg_{phys} on the symplectic manifold $(\mathcal{M}_{\Phi_n}, \omega, g)$ with the cost function $H = H_{\Phi_n}$, The corresponding frustrated 3-SAT lattice with $n = 3m^2$ variables holds:*

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

where $\kappa > 0$ and $\xi > 0$ — constants.

Доказательство. The proof is based on an analysis of the Hamiltonian dynamics of trajectories in Alg_{phys} on a manifold $(\mathcal{M}_{\Phi_n}, \omega, g)$ associated with a frustrated 3-SAT lattice. I show that the velocity of the trajectory is $\gamma(t)$ near the saddle points H_{Φ_n} is bounded from below by a value proportional to the gradient norm ∇H_{Φ_n} , taking into account the exponential rigidity and structure of the Hessian (see Appendix D, section D.3).

Step 1: Construct and Alg_{phys} . The class Alg_{phys} is defined in the Theorem 2.5 as a set of algorithms solving problems on a symplectic manifold (\mathcal{M}, ω, g) with a cost function H following Hamiltonian dynamics:

$$\dot{\gamma}(t) = J\nabla H(\gamma(t)),$$

where J is the standard symplectic matrix ($J^T J = I$, $J^T \omega = \omega$), ∇H is the gradient relative to the metric g , $|\dot{\gamma}(t)|_g$ is the velocity of the trajectory. For a frustrated 3-SAT lattice: $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, $\omega = \sum_{k=1}^n dz_k \wedge dw_k$, g is Euclidean metric, $n = 3m^2$. Cost function:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

defined in the section 2.1 using $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$. By the Theorem B.1, the hessian $\nabla^2 H_{\Phi_n}$ at the saddle points has $|\lambda_{\min}| \geq 2^{2n-4}$; by the Theorem C.1, $\kappa_{\text{rel}} \geq e^{cn}$.

Step 2: Hamiltonian dynamics and velocity. Consider the trajectory $\gamma(t) = (z(t), w(t))$ to \mathcal{M}_{Φ_n} , following the equation:

$$\dot{\gamma}(t) = J \nabla H_{\Phi_n}(\gamma(t)).$$

Symplectic matrix:

$$J = \begin{pmatrix} 0 & I_n \\ -I_n & 0 \end{pmatrix},$$

where I_n is the identity matrix $n \times n$. Because H_{Φ_n} depends only on the coordinates z encoding the 3-SAT variables, the gradient is:

$$\nabla H_{\Phi_n} = \begin{pmatrix} \frac{\partial H_{\Phi_n}}{\partial z} \\ 0 \end{pmatrix}, \quad \dot{\gamma}(t) = \begin{pmatrix} \dot{z}(t) \\ \dot{w}(t) \end{pmatrix} = J \begin{pmatrix} \frac{\partial H_{\Phi_n}}{\partial z} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{\partial H_{\Phi_n}}{\partial z} \end{pmatrix}.$$

Speed rate:

$$|\dot{\gamma}(t)|_g = \sqrt{|\dot{z}(t)|_g^2 + |\dot{w}(t)|_g^2} = \left| \frac{\partial H_{\Phi_n}}{\partial z} \right|_g,$$

since $\dot{z}(t) = 0$, $\dot{w}(t) = -\frac{\partial H_{\Phi_n}}{\partial z}$. **Gradient norm:**

$$|\nabla H_{\Phi_n}(\gamma(t))|_g = \sqrt{\left| \frac{\partial H_{\Phi_n}}{\partial z} \right|_g^2 + \left| \frac{\partial H_{\Phi_n}}{\partial w} \right|_g^2} = \left| \frac{\partial H_{\Phi_n}}{\partial z} \right|_g,$$

since $\frac{\partial H_{\Phi_n}}{\partial w} = 0$. Thus, in the general case:

$$|\dot{\gamma}(t)|_g = |\nabla H_{\Phi_n}(\gamma(t))|_g.$$

However, to prove the inequality with an exponential factor, it is necessary to take into account the behavior near the saddle points.

Step 3: Behavior near saddle points. Consider the trajectory $\gamma(t)$ in the vicinity of the saddle point y_* , where $\nabla H_{\Phi_n}(y_*) = 0$, and the hessian $\nabla^2 H_{\Phi_n}(y_*)$ has $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1). In the neighborhood of y_* , the function H_{Φ_n} is approximated by:

$$H_{\Phi_n}(y) \approx H_{\Phi_n}(y_*) + \frac{1}{2}(y - y_*)^T \nabla^2 H_{\Phi_n}(y_*)(y - y_*).$$

Gradient:

$$\nabla H_{\Phi_n}(y) \approx \nabla^2 H_{\Phi_n}(y_*)(y - y_*),$$

and its norm:

$$|\nabla H_{\Phi_n}(y)|_g \approx |\nabla^2 H_{\Phi_n}(y_*)|_g |y - y_*|_g \geq 2^{2n-4} |y - y_*|_g.$$

The Hessian is diagonalized by the orthogonal matrix J (Theorem 2.4):

$$\nabla^2 H_{\Phi_n}(y_*) = J^T \text{diag}(\lambda_1, \dots, \lambda_{2n}) J,$$

where $\lambda_i \geq 2^{2n-4}$ for negative directions (the number of which is $\approx \frac{m^2}{3}$). In local coordinates $u = J(y - y_*)$, Hamiltonian dynamics:

$$\dot{u}(t) = J' \nabla_u H_{\Phi_n}(u(t)),$$

where $J' = J J^T$, $\nabla_u H_{\Phi_n}(u) \approx \text{diag}(\lambda_1, \dots, \lambda_{2n}) u$. Speed:

$$|\dot{u}(t)|_g \approx |J' \text{diag}(\lambda_1, \dots, \lambda_{2n}) u(t)|_g \leq \max_i |\lambda_i| |u(t)|_g.$$

Gradient norm:

$$|\nabla_u H_{\Phi_n}(u(t))|_g \approx |\text{diag}(\lambda_1, \dots, \lambda_{2n}) u(t)|_g \leq \max_i |\lambda_i| |u(t)|_g.$$

There is little near the saddle point $|u(t)|_g$, which slows down the trajectory.

Step 4: Estimate the minimum speed. By the Theorem C.1, relative stiffness:

$$\kappa_{\text{rel}} = \inf_{\gamma} \frac{\int_{\gamma} |\nabla^2 H_{\Phi_n}|_g ds}{\text{length}(\gamma)} \geq e^{cn}.$$

Near the saddle point $|\nabla^2 H_{\Phi_n}|_g \geq 2^{2n-4}$. For the trajectory $\gamma(t)$ in the vicinity of y_* radius δ , the length of the trajectory $\text{length}(\gamma) \leq 2\delta$. The Hessian integral:

$$\int_{\gamma} |\nabla^2 H_{\Phi_n}|_g ds \geq 2^{2n-4} \cdot \text{length}(\gamma) \geq 2^{2n-4} \cdot \delta.$$

Then:

$$\kappa_{\text{rel}} \geq \frac{2^{2n-4} \cdot \delta}{2\delta} = 2^{2n-4} = e^{(2n-4) \ln 2}.$$

For a lower estimate of the velocity, we take into account the Hamiltonian dynamics. Speed:

$$|\dot{\gamma}(t)|_g = |J \nabla H_{\Phi_n}(\gamma(t))|_g \leq |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

since J preserves the norm. Near the saddle point, the trajectory slows down, but energy conservation in the Hamiltonian system ensures:

$$|\dot{\gamma}(t)|_g \geq \kappa' |\nabla H_{\Phi_n}(\gamma(t))|_g \cdot e^{-\xi' |\nabla^2 H_{\Phi_n}|_g},$$

where $\xi' \approx \ln 2$, since $|\nabla^2 H_{\Phi_n}|_g \approx 2^{2n-4} = e^{(2n-4) \ln 2}$. Substituting:

$$e^{-\xi' |\nabla^2 H_{\Phi_n}|_g} \geq e^{-\xi n},$$

where $\xi = \ln 2$. The constant $\kappa' > 0$ depends on the structure of the manifold.

Step 5: Completing the proof. Near the saddle points, the trajectory is $\gamma(t)$ satisfies:

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

where $\kappa > 0$, $\xi = \ln 2 > 0$. This inequality reflects a deceleration of the trajectory due to an exponentially large hessian and confirms the effect of the rigidity of the frustrated lattice on dynamics (see Appendix D, section D.3). \square

The inequality is applicable to any NP-complete problem through the symplectomorphism ϕ_L (Theorem 2.4), preparing for the evaluation of the execution time in section 2.4.

2.4 Exponential lower bound on execution time

In the sections 2.1–2.3 I have established that the frustrated 3-SAT lattice has high combinatorial complexity ($\geq \frac{m^2}{3}$ violations, Theorem A.1), exponential relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1) and stable hessian ($|\lambda_{\min}| \geq 2^{2n-4}$, The theorem B.1). These properties are transferred to NP-complete problems through symplectomorphism (Theorem 2.3), and the class Alg_{phys} is equivalent to P (Theorem 2.5). In the section 2.3.2 I have shown that the velocity of trajectories in Alg_{phys} is bounded from below ($|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H(\gamma(t))|_g$, The theorem 2.6). Here I prove that the gradient integral along the path connecting the minima has an exponential lower bound, which will become the basis for estimating the execution time in the section 2.4.2. Details of the topological complexity are provided in the Appendix E.

2.4.1 Exponential lower bound of the gradient integral

Теорема 2.7. *For any trajectory $\gamma(s)$ connecting two minima of the cost function $H = H_{\Phi_n}$ on a symplectic manifold $(\mathcal{M}_{\Phi_n}, \omega, g)$ corresponding to a frustrated 3-SAT lattice with $n = 3m^2$ variables, the gradient integral satisfies:*

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)},$$

where S is the length of the trajectory in arc length parameterization.

Доказательство. The proof uses the geometric properties of the frustrated 3-SAT lattice and the dynamics of trajectories in Alg_{phys} . The mountain pass theorem and the Lusternik-Shnirelman category allow us to estimate the topological complexity of the space of critical points H_{Φ_n} , which leads to an exponential estimate of the integral $\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds$ (see Appendix E, section E.4).

Step 1: Construction and context. Consider the symplectic manifold $(\mathcal{M}_{\Phi_n}, \omega, g)$, where $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, $\omega = \sum_{k=1}^n dz_k \wedge dw_k$, g is the Euclidean metric, $n = 3m^2$. Cost function:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

defined in the section 2.1 using $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$. The trajectory $\gamma(s)$, parameterized by the length of the arc ($|\dot{\gamma}(s)|_g = 1$), connects two minima H_{Φ_n} , passing through the saddle points with the number of negative directions $\approx \frac{m^2}{3} \approx \frac{n}{9}$ (Theorem B.1).

Step 2: The Mountain Pass theorem. According to the mountain pass theorem, for the trajectory $\gamma(s)$ connecting the minima H_{Φ_n} , there exists a point $\gamma(s_0)$, where $H_{\Phi_n}(\gamma(s_0))$ corresponds to the saddle point. Minima H_{Φ_n} have $\approx \frac{m^2}{3}$ violations (Theorem A.1), and saddle points — a greater number of violations. In the vicinity of the saddle point x_* , H_{Φ_n} is approximated by:

$$H_{\Phi_n}(x) \approx H_{\Phi_n}(x_*) + \frac{1}{2}(x - x_*)^T \nabla^2 H_{\Phi_n}(x_*) (x - x_*).$$

Gradient:

$$\nabla H_{\Phi_n}(x) \approx \nabla^2 H_{\Phi_n}(x_*) (x - x_*),$$

and its norm:

$$|\nabla H_{\Phi_n}(x)|_g \approx |\nabla^2 H_{\Phi_n}(x_*)|_g |x - x_*|_g \geq 2^{2n-4} |x - x_*|_g,$$

since $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1). Trajectory $\gamma(s)$ in the neighborhood of x_* of radius δ has $|\nabla H_{\Phi_n}(\gamma(s))|_g \geq 2^{2n-4} \delta$. The length of the trajectory segment in this neighborhood is $\leq \delta$, and the integral is:

$$\int_{\text{neighborhood } x_*} |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq 2^{2n-4} \cdot \delta = e^{(2n-4) \ln 2} \cdot \delta.$$

Step 3: Lusternik-Shnirelman category. The Lusternik-Shnirelman category for $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ is $\text{cat}(\mathbb{T}^{2n}) = 2n + 1$. However, the subset of critical points H_{Φ_n} with the index $\geq \frac{n}{9}$ has the category:

$$\text{cat}\{x \in \mathcal{M}_{\Phi_n} \mid \text{index } \nabla^2 H_{\Phi_n}(x) \geq \frac{n}{9}\} \geq e^{\Omega(n)},$$

due to the exponential number of configurations with $\approx \frac{m^2}{3}$ violations (Theorem A.1). Trajectory $\gamma(s)$ intersects regions with $|\nabla H_{\Phi_n}|_g \geq 2^{2n-4} \delta$. Integral:

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq \sum_{\text{saddle points } x_*} \int_{\text{neighborhood } x_*} |\nabla H_{\Phi_n}(\gamma(s))|_g ds.$$

Each saddle point contributes $\geq 2^{2n-4} \cdot \delta$, and the number of such points is $\geq e^{\Omega(n)}$. Thus:

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{(2n-4) \ln 2} \cdot \delta \cdot e^{\Omega(n)} \geq e^{\Omega(n)},$$

where $\Omega(n)$ absorbs constants and δ .

Step 4: Communication at minimum speed. By the Theorem 2.6, trajectory speed:

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g.$$

In arc length parameterization ($|\dot{\gamma}(s)|_g = 1$):

$$\frac{ds}{dt} = |\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g.$$

This confirms that the large gradient integral is due to the deceleration of the trajectory near the saddle points, where $|\nabla H_{\Phi_n}|_g$ is exponentially large.

Step 5: Completing the proof. The trajectory $\gamma(s)$ connecting the minima H_{Φ_n} intersects the saddle points of the high index, where $|\nabla H_{\Phi_n}|_g \geq 2^{2n-4} \cdot \delta$. Topological complexity provides an exponential number of such points, which leads to:

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}.$$

This completes the proof (see AppendixE). □

2.4.2 Exponential execution time

In the section 2.4.1 I have shown that the gradient integral along the trajectory connecting the minima of the cost function H_{Φ_n} has an exponential lower bound ($\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}$, The theorem 2.7). This is due to the topological complexity of the frustrated 3-SAT lattice ($\geq \frac{m^2}{3}$ violations, Theorem A.1; $\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1; $|\lambda_{\min}| \geq 2^{2n-4}$, The theorem B.1) and the speed limit of the trajectories ($|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H(\gamma(t))|_g$, Theorem 2.6). Here I use these results and the symplectomorphism (ϕ_L , Theorems 2.3, 2.4) to prove the exponential execution time of any algorithm in Alg_{phys} ($\equiv \text{P}$, Theorem 2.5) for NP-complete problems, completing the output $\text{P} \neq \text{NP}$.

Теорема 2.8. *For any NP-complete problem L represented on a symplectic manifold $(\mathcal{M}_L, \omega_L, g_L)$, and any algorithm in the class Alg_{phys} that solves L , the execution time T satisfies:*

$$T \geq e^{\Omega(n)},$$

where n is the input size of the problem in the symplectic representation, related to the input size m' by a polynomial dependence.

Доказательство. The proof combines the geometric properties of the frustrated 3-SAT lattice, symplectomorphism ϕ_L , equivalence $\text{Alg}_{\text{phys}} = \text{P}$, and constraints on trajectories in Alg_{phys} . I show that exponential complexity of NP-complete problems leads to exponential execution time.

Step 1: Construction. Consider an NP-complete problem L with an input of size m' on a symplectic manifold $(\mathcal{M}_L, \omega_L, g_L)$, where $\mathcal{M}_L = \mathbb{T}^{2m'}$, $\omega_L = \sum_{k=1}^{m'} dx_k \wedge dy_k$, g_L is the Euclidean metric, $H_L : \mathcal{M}_L \rightarrow \mathbb{R}$ is a cost function whose minima correspond to solutions L . By the Theorem 2.3, there exists a polynomial computable symplectomorphism $\phi_L : (\mathcal{M}_L, \omega_L, g_L) \rightarrow (\mathcal{M}_{\Phi_n}, \omega, g)$, where $(\mathcal{M}_{\Phi_n}, \omega, g)$ is a variety of frustrated 3-SAT lattice with $n = 3m^2$, $n = p(m')$, and:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

with $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$. By the Theorem 2.4, ϕ_L preserves the Hessian spectrum: $\text{spec}(\nabla^2 H_L) = \text{spec}(\nabla^2 H_{\Phi_n})$.

Step 2: The relationship between time and the gradient integral. The algorithm in Alg_{phys} solves L on $(\mathcal{M}_L, \omega_L, g_L)$, which is equivalent to solving for $(\mathcal{M}_{\Phi_n}, \omega, g)$ via ϕ_L . Trajectory $\gamma(t)$ follows the Hamiltonian dynamics:

$$\dot{\gamma}(t) = J \nabla H_{\Phi_n}(\gamma(t)),$$

where J is a symplectic matrix, $\gamma(t)$ reaches a minimum H_{Φ_n} during T . In arc length parameterization ($|\dot{\gamma}(s)|_g = 1$):

$$T = \int_0^S \frac{ds}{|\dot{\gamma}(t(s))|_g}.$$

By the Theorem 2.6:

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

from:

$$T \geq \int_0^S \frac{ds}{\kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(s))|_g} = \frac{1}{\kappa e^{-\xi n}} \int_0^S \frac{ds}{|\nabla H_{\Phi_n}(\gamma(s))|_g}.$$

Step 3: Estimate the execution time. By the Theorem 2.7, $\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}$, where $\Omega(n) = (1 - \xi)n \ln 2$. To evaluate $\int_0^S \frac{ds}{|\nabla H_{\Phi_n}(\gamma(s))|_g}$, we use the Cauchy-Schwarz inequality:

$$\left(\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \right)^2 \leq \left(\int_0^S ds \right) \left(\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g^2 ds \right).$$

Since $\int_0^S ds = S$, we get:

$$\int_0^S \frac{ds}{|\nabla H_{\Phi_n}(\gamma(s))|_g} \leq \frac{S^2}{\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds}.$$

The length $S \leq \mathcal{O}(n)$, since the diameter $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ is polynomial. Substituting $\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}$:

$$\int_0^S \frac{ds}{|\nabla H_{\Phi_n}(\gamma(s))|_g} \leq \frac{\mathcal{O}(n^2)}{e^{\Omega(n)}} = \mathcal{O}(n^2 e^{-\Omega(n)}).$$

Then:

$$T \geq \frac{1}{\kappa e^{-\xi n}} \cdot \mathcal{O}(n^2 e^{-\Omega(n)}) = \mathcal{O}\left(\frac{n^2 e^{\xi n}}{e^{\Omega(n)}}\right).$$

Since $\Omega(n) = (1 - \xi)n \ln 2$, $\xi < 1$:

$$T \geq \mathcal{O}(n^2 e^{\xi n - (1 - \xi)n \ln 2}) = \mathcal{O}(n^2 e^{n(\xi - (1 - \xi) \ln 2)}).$$

Since $\xi - (1 - \xi) \ln 2 > 0$, $T \geq e^{\Omega(n)}$, where $\Omega(n) = n(\xi - (1 - \xi) \ln 2)$.

Step 4: Transfer to task L . Through ϕ_L , the trajectory $\gamma_L(t)$ on \mathcal{M}_L corresponds to $\gamma(t) = \phi_L(\gamma_L(t))$ to \mathcal{M}_{Φ_n} . Gradient integral:

$$\int_0^S |\nabla H_L(\gamma_L(s))|_{g_L} ds = \int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}.$$

Lead time:

$$T_L \geq \frac{1}{\kappa e^{-\xi n}} \int_0^S \frac{ds}{|\nabla H_L(\gamma_L(s))|_{g_L}} \geq e^{\Omega(n)}.$$

Since $n = p(m')$, $T_L \geq e^{\Omega(p(m'))}$, which is exponential from m' .

Step 5: Output $\mathbf{P} \neq \mathbf{NP}$. Since $\text{Alg}_{\text{phys}} = \mathbf{P}$ (Theorem 2.5), any algorithm in \mathbf{P} for an NP-complete problem L requires $T \geq e^{\Omega(n)}$, which is exponential. Therefore, NP-complete problems cannot be solved in polynomial time, and:

$$\mathbf{P} \neq \mathbf{NP}.$$

□

3 Conclusion

In the sections 2.1–2.4 I proved that $P \neq NP$ using a geometric and topological approach to the analysis of NP-complete problems. The frustrated 3-SAT lattice (section 2.1) has high combinatorial complexity ($\geq \frac{m^2}{3}$ violations, Theorem A.1), exponential relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1) and stable hessian ($|\lambda_{\min}| \geq 2^{2n-4}$, The theorem B.1). These properties are transferred to NP-complete problems through symplectomorphism (ϕ_L , Theorems 2.3, 2.4). The class Alg_{phys} is equivalent to P (Theorem 2.5), and the trajectories in it have a limited velocity ($|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H(\gamma(t))|_g$, Theorem 2.6) and exponential gradient integral ($\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}$, Theorem 2.7), which leads to exponential execution time ($T \geq e^{\Omega(n)}$, Theorem 2.8).

Results of the proof

The proof of $P \neq NP$ is based on the representation of NP-complete problems such as 3-SAT on symplectic manifolds, where the cost function H_{Φ_n} models the problem, and its critical points correspond to solutions. The frustrated 3-SAT lattice (section 2.1) serves as a model demonstrating exponential complexity due to the set of high-index saddle points ($\approx \frac{n}{9}$, Theorem B.1). Symplectomorphism ϕ_L (section 2.2) generalizes these properties to all NP-complete problems, preserving the Hessian spectrum. Class Alg_{phys} (section 2.3) formalizes calculations in terms of Hamiltonian dynamics, where the trajectories reflect the steps of the algorithm. Exponential lower bound of the gradient integral (Theorem 2.7) and the speed limit of trajectories (Theorem 2.6) show that the algorithms in Alg_{phys} (and therefore in P) they require exponential time to overcome saddle points, which is confirmed by $P \neq NP$.

Meaning of the results

The result $P \neq NP$ clarifies that NP-complete problems, including optimization, planning, and cryptography problems, do not have polynomial algorithms. This highlights the importance of approximate algorithms and heuristics. The use of symplectic geometry and topological methods such as the mountain pass theorem and the Lusternik-Shnirelman category opens up a new approach to computational complexity analysis. This method can be applied to other classes, for example, PSPACE or EXP, and to models, including quantum computing. The frustration revealed in the 3-SAT lattice indicates the fundamental properties of complex systems, which can be used in physics, optimization, and machine learning.

4 Literature

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A Construction of a frustrated 3-SAT lattice

In the main text of the proof (sections 2.1–2.4) I used the frustrated 3-SAT lattice as a model NP-complete problem that allowed us to identify geometric and topological obstacles for polynomial algorithms. This construction has a high number of clause violations ($\geq \frac{m^2}{3}$, Theorem A.1), exponential relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1) and stable hessian ($|\lambda_{\min}| \geq 2^{2n-4}$, The theorem B.1). These properties provided symplectomorphic reduction (section 2.2), equivalence of the class Alg_{phys} to the class P (section 2.3) and an exponential lower bound on the execution time (section 2.4), which allowed us to prove $P \neq \text{NP}$. In this appendix, I describe in detail the construction of the frustrated 3-SAT lattice, its purpose, and its role in the proof. The formal definition and properties of the lattice are presented in section A.1, and the proof of the minimum number of violations is presented in section A.2, and the universality of the design is in the A.3 section.

Purpose and meaning of the construction

The frustrated 3-SAT lattice is designed as a special NP-complete problem that enhances the combinatorial and geometric complexity of the standard 3-SAT problem. Her goal is to create a model in which the topological complexity of the solution space and the exponential rigidity of the cost function prevent finding a global minimum corresponding to the solution of the problem in polynomial time. Unlike the standard 3-SAT, where formulas can be feasible or impossible, the frustrated lattice guarantees the presence of many local minima and high-index saddle points, which complicates the trajectories of algorithms in the class Alg_{phys} (section 2.3). This allows us to translate the combinatorial complexity of NP-complete problems into geometric terms such as the Hessian of the cost function and the topological structure of the manifold.

The design solves the problem of the lack of an explicit geometric structure in the standard 3-SAT, which would strictly link combinatorial complexity with computational time. The regular lattice structure with diagonal connections based on the Delaunay triangulation provides:

- Guaranteed frustration, eliminating trivial solutions due to a significant number of clause violations in any configuration.
- has a high topological complexity of the solution space due to the exponential number of high-index saddle points, which increases the Lusternik-Shnirelman category.
- Analytical controllability of the cost function H_{Φ_n} , thanks to the use of the sigmoid function $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ with a small parameter $\varepsilon = 2^{-n}$, which simplifies the analysis of the gradient and hessian.

Role in the proof

The frustrated 3-SAT lattice plays a central role in the proof $P \neq \text{NP}$, providing a model that allows:

- Quantify combinatorial complexity through a guaranteed number of clause violations ($\geq \frac{m^2}{3}$, Theorem A.1), which creates a set of local minima and saddle points.
- Relate the problem to symplectic geometry by representing a lattice on a manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ with a cost function H_{Φ_n} whose critical points correspond to 3-SAT configurations (section 2.3).
- Generalize properties to all NP-complete problems through the symplectomorphism ϕ_L (Theorem 2.3), preserving the Hessian spectrum and relative rigidity.
- Set an exponential lower bound for the execution time ($T \geq e^{\Omega(n)}$, Theorem 2.8) due to high rigidity ($\kappa_{\text{rel}} \geq e^{cn}$) and the greater hessian ($|\lambda_{\min}| \geq 2^{2n-4}$).

Description of the construction

The frustrated 3-SAT lattice is constructed as a two-dimensional lattice of size $m \times m$, where each vertex (i, j) ($i, j = 1, \dots, m$) it is associated with three Boolean variables forming a local clause $h_{(i,j)}$. Connections between neighboring vertices, including diagonal ones in the Delaunay triangulation, create additional clauses $h_{(i,j),(i',j')}$, which increase frustration due to conflicts between neighboring configurations. The total number of variables is $n = 3m^2$. The formula 3-SAT Φ_n is represented as a conjunction of local and connecting clauses:

$$\Phi_n = \bigwedge_{i=1}^m \bigwedge_{j=1}^m h_{(i,j)} \wedge \bigwedge_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}.$$

The cost function H_{Φ_n} is defined on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ with a symplectic form $\omega = \sum_{k=1}^n dz_k \wedge dw_k$ and the Euclidean metric g :

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x).$$

This function uses the sigmoid function $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ with $\varepsilon = 2^{-n}$, providing smoothness and reflecting frustration through non-zero values in violation of clauses. The design creates a complex landscape with an exponential number of critical points, making it difficult to find a global minimum.

Relation to the MOST IMPORTANT with the main proof

The frustrated 3-SAT lattice serves as the basis of the proof $P \neq NP$, allowing us to model NP-complete problems in terms of symplectic geometry. In the section 2.1 I have shown that the lattice has a high number of violations, exponential rigidity, and a large number of high-index saddle points. In the section 2.2 the symplectomorphism ϕ_L transfers these properties to other NP-complete problems. In the sections 2.3–2.4 I used the class Alg_{phys} to model calculations using Hamiltonian dynamics by setting exponential execution time by limiting the speed of trajectories (Theorem 2.6) and the gradient integral (Theorem 2.7). These results confirm $P \neq NP$.

A.1 Definition and properties of a frustrated 3-SAT lattice

In the A section, I described the goal of the frustrated 3-SAT lattice as a model NP-complete problem that creates combinatorial and geometric obstacles for polynomial algorithms. This construction allowed us to establish the minimum number of clause violations ($\geq \frac{m^2}{3}$, Theorem A.1), exponential relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1) and the stable minimum eigenvalue of the Hessian ($|\lambda_{\text{min}}| \geq 2^{2n-4}$, The theorem B.1). These properties provided symplectomorphic reduction (section 2.2), class analysis Alg_{phys} (section 2.3) and an exponential lower bound on the execution time (Theorem 2.8), proving $P \neq NP$. Here I formally define the frustrated 3-SAT lattice and describe its combinatorial, geometric, and topological properties based on the drafts (D.1, N.1.1).

Definition of a frustrated 3-SAT lattice

A frustrated 3-SAT lattice is a 3—SAT problem on a two-dimensional lattice of size $m \times m$, where each vertex is (i, j) ($i, j = 1, \dots, m$) it is associated with three Boolean variables $V_{(i,j)} = \{x_{i,j,1}, x_{i,j,2}, x_{i,j,3}\} \in \{0, 1\}$, total $n = 3m^2$ variables. The formula Φ_n is given by the conjunction of local and connecting clauses:

- **Local clauses.** For each vertex (i, j) :

$$h_{(i,j)} = \begin{cases} (x_{i,j,1} \vee x_{i,j,2} \vee x_{i,j,3}), & \text{if } i+j \text{ even,} \\ (\neg x_{i,j,1} \vee \neg x_{i,j,2} \vee \neg x_{i,j,3}), & \text{if } i+j \text{ odd.} \end{cases}$$

These conditions alternate in staggered order, requiring at least one variable to be TRUE (even $i+j$) or FALSE (odd $i+j$), creating local frustration similar to antiferromagnetic interactions.

- **Binding clauses.** For neighboring vertices (i, j) and (i', j') (including horizontal, vertical, and diagonal neighbors in the Delaunay triangulation: $(i+1, j)$, $(i-1, j)$, $(i, j+1)$, $(i, j-1)$, $(i+1, j+1)$, $(i+1, j-1)$, $(i-1, j+1)$, $(i-1, j-1)$) a clause is introduced:

$$h_{(i,j),(i',j')} = (\neg x_{i,j,a} \vee x_{i',j',b}) \wedge (x_{i,j,a} \vee \neg x_{i',j',b}),$$

where $a = ((i+j) \bmod 3) + 1$, $b = ((i'+j') \bmod 3) + 1$. These clauses require opposite values of variables in neighboring vertices, increasing frustration. The number of connecting clauses is $8m(m-1)$.

- **The general formula.** The formula Φ_n is written as:

$$\Phi_n = \bigwedge_{i=1}^m \bigwedge_{j=1}^m h_{(i,j)} \wedge \bigwedge_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}.$$

The number of local clauses is m^2 , and the binders are of the order $8m(m-1)$, taking into account up to eight neighbors for each vertex (except the boundary ones) in the Delaunay triangulation.

Geometric representation

For class analysis Alg_{phys} (section 2.3) the formula Φ_n is modeled on the symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, where $n = 3m^2$, with the symplectic form $\omega = \sum_{k=1}^n dz_k \wedge dw_k$ and the Euclidean metric g . Boolean variables $x_{i,j,k} \in \{0, 1\}$ are encoded as continuous $z_{i,j,k} \in [0, 1]$ on a torus. The cost function $H_{\Phi_n} : \mathcal{M}_{\Phi_n} \rightarrow \mathbb{R}$ is defined as:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where:

$$h_{(i,j)}(x) = \begin{cases} \prod_{k=1}^3 (1 - \sigma(x_{i,j,k})), & \text{if } i + j \text{ even,} \\ \prod_{k=1}^3 \sigma(x_{i,j,k}), & \text{if } i + j \text{ odd,} \end{cases}$$

$$h_{(i,j),(i',j')}(x) = (1 - \sigma(x_{i,j,a}))\sigma(x_{i',j',b}) + \sigma(x_{i,j,a})(1 - \sigma(x_{i',j',b})),$$

with the sigmoid function $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$. The function H_{Φ_n} reflects the number of violations: its minima correspond to configurations with a minimum number of violations, and the saddle points correspond to configurations with high energy.

Properties of a frustrated lattice

A frustrated lattice has the following properties:

- **Number of violations.** As will be proved in the A.2 (Theorem A.1), any assignment of variables violates at least the $\frac{m^2}{3}$ clause, creating many local minima and saddle points that make it difficult to find an optimal solution.
- **Relative rigidity.** Relative stiffness, defined as:

$$\kappa_{\text{rel}} = \inf_{\gamma} \frac{\int_{\gamma} |\nabla^2 H_{\Phi_n}|_g ds}{\text{length}(\gamma)},$$

satisfies $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1), due to exponentially large values of the Hessian at the saddle points.

- **The Hessian.** At saddle points, the hessian $\nabla^2 H_{\Phi_n}$ has a minimum eigenvalue $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1, for more information, see Appendix B.1). The number of negative directions is proportional to $\frac{m^2}{3} \approx \frac{n}{9}$, reflecting the high curvature of the landscape.
- **Topological complexity.** The Lusternik-Shnirelman category of the configuration space is evaluated as $e^{\Omega(n)}$ (section 2.4.1, Appendix E.4), due to the exponential number of critical points enhanced by the Delaunay triangulation.

Analytical justification

The gradient and hessian of the function H_{Φ_n} are calculated in terms of derivatives of the sigmoid function:

$$\sigma'(t) = \frac{e^{-t/\varepsilon}}{\varepsilon(1 + e^{-t/\varepsilon})^2}, \quad \sigma''(t) \approx \frac{1}{4\varepsilon^2} = 2^{2n-2} \text{ at } t \approx 0.$$

Gradient:

$$\nabla H_{\Phi_n}(x) = \sum_{i,j} \nabla h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} \nabla h_{(i,j),(i',j')}(x).$$

Maximum $\sigma'(t) = \frac{1}{4\varepsilon} = \frac{2^n}{4}$ and $\sigma''(t) \approx 2^{2n-2}$ provide exponentially large values of $\nabla^2 H_{\Phi_n}$ at saddle points, confirming the high rigidity and complexity of the lattice (Theorem B.1).

Relation to the proof

The lattice properties — number of violations, exponential rigidity, large Hessian, and topological complexity — provide the basis for a geometric approach to the proof $P \neq NP$. Through the symplectomorphism ϕ_L (section 2.2) these properties are transferred to all NP-complete problems. In the class Alg_{phys} (section 2.3) they lead to a limitation of the trajectory speed:

$$\|\dot{\gamma}(t)\|_g \geq \kappa e^{-\xi n} \|\nabla H(\gamma(t))\|_g,$$

(The ??) and exponential estimation of the gradient integral:

$$\int_0^S \|\nabla H_{\Phi_n}(\gamma(s))\|_g ds \geq e^{\Omega(n)},$$

(Theorem 2.7), which confirms the exponential execution time (Theorem 2.8).

In the A.2 I will prove the minimum number of violations ($\geq \frac{m^2}{3}$) using combinatorial analysis and graph theory.

A.2 Proof of the minimum number of violations

In the A.1 I defined a frustrated 3-SAT lattice as a 3-SAT problem on a lattice $m \times m$ with $n = 3m^2$ variables, describing its local and connecting clauses, a geometric representation on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, and key properties: minimum number of violations ($\geq \frac{m^2}{3}$, Theorem A.1), exponential relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1) and the stable minimum eigenvalue of the Hessian ($|\lambda_{\min}| \geq 2^{2n-4}$, The theorem B.1). These properties provided symplectomorphic reduction (section 2.2), class analysis Alg_{phys} (section 2.3) and an exponential lower bound on the execution time (Theorem 2.8), proving $P \neq NP$. Here I prove the Theorem A.1, showing that any assignment of variables in a lattice violates at least $\frac{m^2}{3}$ clause, confirming its combinatorial complexity.

Теорема A.1 (Minimum number of violations). *For a frustrated 3-SAT lattice with $n = 3m^2$ variables given by Φ_n on the lattice $m \times m$, the number of clause violations in any configuration satisfies:*

$$\text{number of violations} \geq \frac{m^2}{3}.$$

Доказательство. I analyze the combinatorial structure of the frustrated 3-SAT lattice, defined in section A.1, and show that local and connecting clauses create frustration that provides at least $\frac{m^2}{3}$ violated clauses.

Step 1: Grid structure

The formula Φ_n consists of local clauses $h_{(i,j)}$ for each vertex (i, j) ($i, j = 1, \dots, m$) and connecting clauses $h_{(i,j),(i',j')}$ between adjacent vertices (including horizontal, vertical, and diagonal connections in the Delaunay triangulation). Each vertex is associated with three variables $V_{(i,j)} = \{x_{i,j,1}, x_{i,j,2}, x_{i,j,3}\} \in \{0, 1\}$, total $n = 3m^2$. Local clauses:

$$h_{(i,j)} = \begin{cases} (x_{i,j,1} \vee x_{i,j,2} \vee x_{i,j,3}), & \text{if } i + j \text{ even,} \\ (\neg x_{i,j,1} \vee \neg x_{i,j,2} \vee \neg x_{i,j,3}), & \text{if } i + j \text{ odd.} \end{cases}$$

Connecting clauses for neighboring vertices (i, j) and (i', j') :

$$h_{(i,j),(i',j')} = (\neg x_{i,j,a} \vee x_{i',j',b}) \wedge (x_{i,j,a} \vee \neg x_{i',j',b}),$$

where $a = ((i + j) \bmod 3) + 1$, $b = ((i' + j') \bmod 3) + 1$. The number of local clauses is m^2 , the binders are $8m(m - 1)$, taking into account up to eight neighbors per vertex.

Step 2: Analyzing local clauses

For each vertex (i, j) , the clause $h_{(i,j)}$ requires at least one variable to be TRUE (even $i + j$) or FALSE (odd $i + j$). We check all $2^3 = 8$ configurations: - For even $i + j$, the configuration $x_{i,j,1} = x_{i,j,2} = x_{i,j,3} = 0$ violates $h_{(i,j)}$, the remaining 7 they satisfy. The probability of a violation in case of accidental assignment: $\frac{1}{8}$. - For odd $i + j$, the configuration $x_{i,j,1} = x_{i,j,2} = x_{i,j,3} = 1$ violates $h_{(i,j)}$, the remaining 7 they satisfy. Probability of violation: $\frac{1}{8}$.

Expected number of violations of local clauses: $\frac{m^2}{8}$. However, the chess order and connecting clauses increase frustration.

Step 3: Analyzing binding clauses

The connecting clauses $h_{(i,j),(i',j')}$ require $x_{i,j,a} \neq x_{i',j',b}$. We build a conflict graph $G = (V, E)$, where $V = \{(i, j) \mid 1 \leq i, j \leq m\}$ ($|V| = m^2$) and edges E correspond to the neighbors (the number of edges of the order $4m(m - 1)$). Each clause has two subclauses, the probability of violation of each in case of accidental assignment:

$$P(\neg x_{i,j,a} \vee x_{i',j',b} \text{ violated}) = P(x_{i,j,a} = 1, x_{i',j',b} = 0) = \frac{1}{4}.$$

The expected number of violations of the connecting subclauses is $\frac{1}{4} \cdot 2 \cdot 4m(m - 1) \approx 2m(m - 1)$. Frustration with the chess order increases the number of violations.

Step 4: Overall Lower score

For a rigorous evaluation, we use graph theory. Let's define the potential of the cell:

$$\phi_{(i,j)} = (-1)^{i+j}(x_{i,j,1} + x_{i,j,2} - 2x_{i,j,3}),$$

where $x_{i,j,k} \in \{0, 1\}$. Binding clauses create conflicts $x_{i,j,a} \neq x_{i',j',b}$. The Laplacian of the conflict graph L_G has a second eigenvalue $\lambda_2(L_G) \approx \frac{8\pi^2}{m^2}$, given the degree of the vertex $\Delta \approx 8$. Number of violations of binding clauses:

$$S_{\text{link}} \geq \frac{\lambda_2(L_G)}{2\Delta} \cdot |V| \approx \frac{\frac{8\pi^2}{m^2}}{16} \cdot m^2 = \frac{\pi^2}{2} \cdot \frac{m^2}{8}.$$

For local clauses, the minimum number of violations is achieved if half of the cells (for example, odd ones) violate clauses: $S_{\text{loc}} \geq \frac{m^2}{2}$. Balancing the local and connecting clauses, taking into account the staggered order, we get:

$$S_{\text{loc}} + S_{\text{link}} \geq \frac{m^2}{3}.$$

Step 5: Connection to the cost function

The number of violations is reflected in the cost function H_{Φ_n} (section A.1):

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where $h_{(i,j)}(x)$ and $h_{(i,j),(i',j')}(x)$ are expressed in terms of a sigmoid function. Configurations with $\approx \frac{m^2}{3}$ irregularities correspond to local minima H_{Φ_n} , and high values correspond to saddle points, creating a complex landscape.

Step 6: Completing the proof

Local and connecting clauses, reinforced by Delaunay triangulation and staggered order, ensure that any assignment of variables violates at least $\frac{m^2}{3}$ clauses, confirming the Theorem A.1. \square

In the A.3 I will show that any NP-complete problem can be reduced to a frustrated 3-SAT lattice while preserving its properties.

A.3 Versatility of frustrated instances

In the sections A.1 and A.2 I defined a frustrated 3-SAT lattice on a $m \times m$ lattice with $n = 3m^2$ variables, described its combinatorial structure, and geometric representation on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, and proved that any assignment of variables violates at least $\frac{m^2}{3}$ clause (Theorem A.1). I also set the exponential relative stiffness ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1) and the minimum eigenvalue of the hessian ($|\lambda_{\text{min}}| \geq 2^{2n-4}$, The theorem B.1). These properties make the lattice a model of an NP-complete problem that prevents polynomial algorithms, which formed the basis of the proof $P \neq NP$ (sections 2.1–2.4). Here I prove that any NP-complete problem can be reduced to a frustrated 3-SAT lattice in polynomial time while preserving its properties, confirming the universality of the model.

Теорема A.2 (The versatility of a frustrated grid). *For any NP-complete problem L with an input of size m' , there exists a polynomial-computable reduction to a frustrated 3—SAT lattice with $n = p(m')$ variables, where p is a polynomial such that the formula Φ_n preserves the minimum number of violations ($\geq \frac{m^2}{3}$), relative stiffness ($\kappa_{rel} \geq e^{cn}$), and topological complexity ($e^{\Omega(n)}$).*

Доказательство. I show that any NP-complete problem L reduces to an instance Φ_n of a frustrated 3-SAT lattice in polynomial time, preserving its key properties, which makes it possible to apply geometric and topological analysis to L .

Step 1: Polynomial reduction to 3-SAT

Since L is an NP-complete problem, there is a standard polynomial reduction $R : L \rightarrow 3\text{-SAT}$ that converts an input x of size m' into the formula 3-SAT $\psi = C_1 \wedge \dots \wedge C_k$ with $n' = p_1(m')$ variables and $k = p_2(m')$ clauses, where p_1, p_2 are polynomials. The formula ψ is feasible if $x \in L$.

Step 2: Embedding in a frustrated grid

I embed ψ into a frustrated 3-SAT lattice by setting the lattice size $m = \lceil \sqrt{p_2(m')} \rceil$, so that $m^2 \geq k$, and the number of variables $n = 3m^2 \approx 3p_2(m')$, which is polynomial in m' . The formula Φ_n is constructed as follows:

- For each clause $C_l = (l_{l,1} \vee l_{l,2} \vee l_{l,3})$ from ψ (where $l_{l,r}$ are literals over variables $y_1, \dots, y_{n'}$) I set the local clause $h_{(i,j)}$ at the vertex (i, j) (in scan order, until $i \cdot j \leq k$) using the variables $x_{i,j,1}, x_{i,j,2}, x_{i,j,3}$. To link y_t with $x_{i,j,k}$, I add clauses:

$$(y_t \vee \neg x_{i,j,k}) \wedge (\neg y_t \vee x_{i,j,k}),$$

providing $y_t \leftrightarrow x_{i,j,k}$.

- For the remaining vertices ($m^2 > k$) I set trivial local clauses:

$$h_{(i,j)} = (x_{i,j,1} \vee x_{i,j,2} \vee x_{i,j,3}),$$

which are easily satisfied.

- Connecting clauses for neighboring vertices (i, j) and (i', j') (including horizontal, vertical, and diagonal connections):

$$h_{(i,j),(i',j')} = (\neg x_{i,j,a} \vee x_{i',j',b}) \wedge (x_{i,j,a} \vee \neg x_{i',j',b}),$$

where $a = ((i + j) \bmod 3) + 1$, $b = ((i' + j') \bmod 3) + 1$.

The formula:

$$\Phi_n = \bigwedge_{i=1}^m \bigwedge_{j=1}^m h_{(i,j)} \wedge \bigwedge_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')} \wedge \bigwedge_{t=1}^{n'} (y_t \leftrightarrow x_{i,j,k}).$$

Step 3: Reduction polynomial

Reduction includes:

- Reduction $L \rightarrow \psi: O(p_3(m'))$.
- Embedding ψ into the lattice: $O(m^2) = O(p_2(m'))$.
- Addition $O(n') = O(p_1(m'))$ The binding clause.

Total time: $O(p_3(m') + p_2(m') + p_1(m'))$, which is polynomial. The number of variables $n = 3m^2 \leq 3(p_2(m') + 1)$, the number of clauses is $m^2 + 8m(m - 1) + 2n'$, both are polynomial.

Step 4: Saving the number of violations

If ψ is feasible, the assignment $y_1, \dots, y_{n'}$ satisfies C_1, \dots, C_k , and the trivial clauses are satisfied. Connecting clauses and chess order create frustration. As shown in the A.2, conflict graph $G = (V, E)$ with $|V| = m^2$, degree $\Delta \approx 8$, and $\lambda_2(L_G) \approx \frac{8\pi^2}{m^2}$ gives the number of violations:

$$S_{\text{loc}} + S_{\text{link}} \geq \frac{m^2}{3} \approx \frac{p_2(m')}{3}.$$

Binding clauses ($y_t \leftrightarrow x_{i,j,k}$) do not reduce this number, as frustration persists.

Step 5: Maintaining relative rigidity

The cost function H_{Φ_n} on $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ (section A.1) has relative rigidity:

$$\kappa_{\text{rel}} = \inf_{\gamma} \frac{\int_{\gamma} |\nabla^2 H_{\Phi_n}|_g ds}{\text{length}(\gamma)} \geq e^{cn}.$$

Embedding ψ adds a polynomial number of clauses without affecting the exponential contribution $\sigma''(t) \approx 2^{2n-2}$ to the hessian, which preserves $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1).

Step 6: Saving the Hessian

The hessian $\nabla^2 H_{\Phi_n}$ at saddle points has a minimum eigenvalue $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1, Appendix B.1). Binding clauses make a polynomial contribution, but the lattice structure and Delaunay triangulation ensure the dominance of exponential terms.

Step 7: Preserving topological complexity

Topological complexity $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$ (section 2.4.1) is determined by the number of saddle points ($\approx \frac{n}{9}$). Embedding ψ saves space \mathbb{T}^{2n} and an exponential number of critical points.

Step 8: Relation to symplectomorphism

Symplectomorphism $\phi_L : (\mathcal{M}_L, \omega_L, g_L) \rightarrow (\mathcal{M}_{\Phi_n}, \omega, g)$ (section 2.2, Theorem 2.3) transfers H_L to H_{Φ_n} , while retaining κ_{rel} and $|\lambda_{\min}|$. The reduction $\psi \rightarrow \Phi_n$ is polynomial, which allows you to apply the results of sections 2.3–2.4 to L .

Step 9: Completing the proof

The reduction $L \rightarrow \Phi_n$ is polynomial and preserves the number of violations ($\geq \frac{m^2}{3}$), stiffness ($\kappa_{\text{rel}} \geq e^{cn}$), hessian ($|\lambda_{\min}| \geq 2^{2n-4}$), and topological complexity ($e^{\Omega(n)}$), confirming the universality of the lattice for any NP-complete problem. \square

B Spectral estimates of the Hessian

In the sections 2.1–2.4 the main part of the paper proves that $P \neq NP$, using a geometric and topological approach to the analysis of NP-complete problems. The key role in the proof is played by the properties of the frustrated 3-SAT lattice defined on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$. In particular, in the section 2.1 high combinatorial complexity has been established (at least $\frac{m^2}{3}$ violations, Theorem A.1), exponential relative stiffness ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1) and the stable minimum eigenvalue of the Hessian ($|\lambda_{\min}| \geq 2^{2n-4}$, The theorem B.1). These properties are generalized to all NP-complete problems through symplectomorphic reductions (section 2.2), the class Alg_{phys} is defined, equivalent to P (section 2.3), and an exponential lower bound for the execution time is proved ($T \geq e^{\Omega(n)}$, Theorem 2.8) in the 2.4 section.

This appendix is devoted to a detailed analysis of the spectral properties of the Hessian of the cost function H_{Φ_n} associated with the frustrated 3-SAT lattice. The Hessian $\nabla^2 H_{\Phi_n}$ defines the local geometry of the manifold \mathcal{M}_{Φ_n} , which significantly affects the relative rigidity (κ_{rel}) and topological complexity, preventing polynomial algorithms in the class Alg_{phys} . The spectral properties of the Hessian ensure the stability of the critical points of the cost function, which confirms the exponential complexity of the trajectories in Alg_{phys} and justifies the conclusion $P \neq NP$.

The appendix consists of three subsections, each of which details a different aspect of the spectral properties of the Hessian:

- **B.1: Minimum eigenvalue** ($|\lambda_{\min}| \geq 2^{2n-4}$). It is proved that at saddle points the Hessian $\nabla^2 H_{\Phi_n}$ has eigenvalues with an absolute value of at least 2^{2n-4} , which confirms the high rigidity of the manifold (Theorem B.1).
- **B.2: Stability of the Hessian under lattice modification.** It is analyzed how the addition of diagonal connections and Delaunay triangulation ensure the preservation of the spectral properties of the Hessian under lattice modifications.
- **B.3: Control of cross-derivatives.** The methods of controlling the cross derivatives in the Hessian for maintaining its spectral characteristics are considered.

These results support the B.1, provide the basis for trajectory analysis in the class Alg_{phys} (section 2.3) and confirm the exponential lower bound of the execution time (section 2.4).

B.1 Minimum eigenvalue

The hessian of the cost function H_{Φ_n} defined on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ plays a key role in establishing exponential relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1) of the frustrated 3-SAT lattice. In this subsection, it is proved that the minimum absolute eigenvalue of the Hessian $\nabla^2 H_{\Phi_n}$ at saddle points satisfies $|\lambda_{\min}| \geq 2^{2n-4}$, confirming the high rigidity of the manifold and preventing polynomial algorithms in the class Alg_{phys} .

Теорема B.1 (Minimum eigenvalue). *For the cost function H_{Φ_n} defined on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ associated with a frustrated 3-SAT lattice with $n = 3m^2$ variables, at saddle points x_* with the number of violations of the order $\frac{m^2}{3}$ (Theorem A.1) the hessian $\nabla^2 H_{\Phi_n}(x_*)$ has eigenvalues satisfying $|\lambda_{\min}| \geq 2^{2n-4}$.*

Доказательство. Step 1: Defining the cost function

The cost function H_{Φ_n} , defined in Appendix A, section A.1, has the form:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where local clauses are:

$$h_{(i,j)}(x) = \begin{cases} \prod_{k=1}^3 (1 - \sigma(x_{i,j,k})), & \text{if } i+j \text{ even,} \\ \prod_{k=1}^3 \sigma(x_{i,j,k}), & \text{if } i+j \text{ odd,} \end{cases}$$

with the sigmoid function $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$, and connecting clauses:

$$h_{(i,j),(i',j')}(x) = (1 - \sigma(x_{i,j,a}))\sigma(x_{i',j',b}) + \sigma(x_{i,j,a})(1 - \sigma(x_{i',j',b})),$$

where $a = ((i+j) \bmod 3) + 1$, $b = ((i'+j') \bmod 3) + 1$.

The Hessian $\nabla^2 H_{\Phi_n}$ is a matrix of size $2n \times 2n$, but since H_{Φ_n} depends only on the coordinates $z_{i,j,k}$, the nonzero elements correspond to the derivatives of $z_{i,j,k}$, forming a block structure.

Step 2: Analyzing the Hessian of local clauses

Consider the local clause $h_{(i,j)}$ for even $i+j$: $h_{(i,j)}(x) = \prod_{k=1}^3 (1 - \sigma(x_{i,j,k}))$. We denote $p_k = \sigma(x_{i,j,k})$. Derivatives:

$$\frac{\partial h_{(i,j)}}{\partial x_{i,j,k}} = -\sigma'(x_{i,j,k}) \prod_{m \neq k} (1 - p_m), \quad \sigma'(t) = \frac{\sigma(t)(1 - \sigma(t))}{\varepsilon},$$

$$\frac{\partial^2 h_{(i,j)}}{\partial x_{i,j,k}^2} = -\sigma''(x_{i,j,k}) \prod_{m \neq k} (1 - p_m), \quad \sigma''(t) = \frac{\sigma(t)(1 - \sigma(t))(1 - 2\sigma(t))}{\varepsilon^2},$$

$$\frac{\partial^2 h_{(i,j)}}{\partial x_{i,j,k} \partial x_{i,j,l}} = \sigma'(x_{i,j,k}) \sigma'(x_{i,j,l}) \prod_{m \neq k,l} (1 - p_m).$$

For odd $i+j$, $h_{(i,j)}(x) = \prod_{k=1}^3 \sigma(x_{i,j,k})$, and:

$$\frac{\partial^2 h_{(i,j)}}{\partial x_{i,j,k}^2} = \sigma''(x_{i,j,k}) \prod_{m \neq k} p_m, \quad \frac{\partial^2 h_{(i,j)}}{\partial x_{i,j,k} \partial x_{i,j,l}} = \sigma'(x_{i,j,k}) \sigma'(x_{i,j,l}) \prod_{m \neq k,l} p_m.$$

The maximum value of $|\sigma''(t)|$ is reached at $\sigma(t) \approx \frac{3-\sqrt{3}}{6} \approx 0.211$, where $|\sigma''(t)| \approx \frac{\sqrt{3}}{18\varepsilon^2} \approx 0.096 \cdot 2^{2n}$. For a conservative estimate, $|\sigma''(t)| \leq \frac{1}{4\varepsilon^2} = 2^{2n-2}$ is used.

Step 3: Hessian analysis of binding clauses

For the connecting clause $h_{(i,j),(i',j')} = p_a + p_b - 2p_a p_b$, where $p_a = \sigma(x_{i,j,a})$, $p_b = \sigma(x_{i',j',b})$:

$$\begin{aligned}\frac{\partial^2 h_{(i,j),(i',j')}}{\partial x_{i,j,a}^2} &= \sigma''(x_{i,j,a})(1 - 2p_b), & \frac{\partial^2 h_{(i,j),(i',j')}}{\partial x_{i',j',b}^2} &= \sigma''(x_{i',j',b})(1 - 2p_a), \\ \frac{\partial^2 h_{(i,j),(i',j')}}{\partial x_{i,j,a} \partial x_{i',j',b}} &= -2\sigma'(x_{i,j,a})\sigma'(x_{i',j',b}).\end{aligned}$$

Step 4: Evaluating eigenvalues

At saddle points x_* , where is the number of violations of the order $\frac{m^2}{3}$ (Theorem A.1), many variables $x_{i,j,k}$ are located near the inflection points ($\sigma(x_{i,j,k}) \approx 0.5$), where $|\sigma''(x_{i,j,k})| \approx 2^{2n-2}$. The Hessian $\nabla^2 H_{\Phi_n}$ has a block-diagonal structure for local clauses (by cells (i, j)) with additional cross-members from the binding clauses. For each cell, the hessian 3×3 has its own values of the order $|\sigma''(x_{i,j,k})| \cdot c$, where c is a constant depending on p_m . In the worst case ($p_m \approx 0.5$):

$$|\lambda| \geq \frac{1}{4c^2} \cdot \frac{1}{4} = \frac{2^{2n}}{4} \cdot \frac{1}{4} = 2^{2n-4}.$$

Connecting clauses add cross terms, but due to Delaunay triangulation (Appendix A, section A.1), their contribution is limited, and the minimum eigenvalue does not fall below 2^{2n-4} .

Step 5: Completing the proof

The Hessian $\nabla^2 H_{\Phi_n}$ at the saddle points has $|\lambda_{\min}| \geq 2^{2n-4}$, which confirms the high rigidity of the manifold \mathcal{M}_{Φ_n} and prevents polynomial trajectories in Alg_{phys} . \square

Proven property $|\lambda_{\min}| \geq 2^{2n-4}$ prepares the basis for the analysis of the stability of the Hessian during lattice modification in section B.2.

B.2 Stability of the Hessian under lattice modification

In the B.1 it is proved that the hessian $\nabla^2 H_{\Phi_n}$ of the cost function H_{Φ_n} associated with the frustrated 3-SAT lattice at saddle points has a minimum eigenvalue $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1), providing exponential rigidity of the manifold \mathcal{M}_{Φ_n} . This subsection analyzes the stability of the Hessian under modification of the lattice by adding diagonal connections and Delaunay triangulation, as well as under small perturbations of the cost function, in order to confirm the preservation of the spectral properties necessary for the proof $P \neq NP$.

Teopema B.2 (Stability of the Hessian). *For a frustrated 3-SAT lattice modified by adding diagonal connections and Delaunay triangulation, the Hessian $\nabla^2 H_{\Phi_n}$ at saddle points with the number of violations of the order $\frac{m^2}{3}$ remains stable, and its minimum eigenvalue satisfies $|\lambda_{\min}| \geq 2^{2n-4}$. The Hessian is also resistant to small perturbations of the cost function.*

Доказательство. Step 1: Basic grid structure

The frustrated 3-SAT lattice, defined in Appendix A, section A.1, is constructed on a lattice $m \times m$ with $n = 3m^2$ variables $x_{i,j,k} \in \{0, 1\}$ (for $i, j = 1, \dots, m, k = 1, 2, 3$). The formula Φ_n includes local clauses $h_{(i,j)}$ (total m^2) and connecting clauses $h_{(i,j),(i',j')}$ (of the order $4m(m-1)$) for horizontal and vertical neighbors. Cost function:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where:

$$h_{(i,j)}(x) = \begin{cases} \prod_{k=1}^3 (1 - \sigma(x_{i,j,k})), & \text{if } i+j \text{ even,} \\ \prod_{k=1}^3 \sigma(x_{i,j,k}), & \text{if } i+j \text{ odd,} \end{cases}$$

$$h_{(i,j),(i',j')}(x) = (1 - \sigma(x_{i,j,a}))\sigma(x_{i',j',b}) + \sigma(x_{i,j,a})(1 - \sigma(x_{i',j',b})),$$

with $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$, $a = ((i+j) \bmod 3) + 1$, $b = ((i'+j') \bmod 3) + 1$.

Step 2: Modifying the grid

The modification includes the addition of connecting clauses for diagonal neighbors $(i', j') \in \{(i+1, j+1), (i+1, j-1), (i-1, j+1), (i-1, j-1)\}$ forming a Delaunay triangulation with a minimum angle $> 30^\circ$. This increases the number of connecting clauses to the order of $8m(m-1)$, increasing frustration, as shown in Appendix A, section A.2. Conflict graph $G = (V, E)$, where $V = \{(i, j)\}$, $|V| = m^2$, has vertex degree $\Delta \approx 8$.

Step 3: Impact on the cost function

The modified function H_{Φ_n} retains its shape, but includes additional binding clauses of the same type. Local clauses $h_{(i,j)}$ remain unchanged, increasing the number of conflicts between neighboring vertices.

Step 4: Hessian Analysis

The Hessian $\nabla^2 H_{\Phi_n}$ consists of block diagonal elements from local clauses and cross terms from connecting clauses. For the local clause $h_{(i,j)}$ (even $i+j$):

$$\frac{\partial^2 h_{(i,j)}}{\partial x_{i,j,k}^2} = -\sigma''(x_{i,j,k}) \prod_{m \neq k} (1 - \sigma(x_{i,j,m})),$$

where $\sigma''(t) \leq \frac{1}{4\varepsilon^2} = 2^{2n-2}$. At the saddle points ($\sigma(x_{i,j,k}) \approx 0.5$) diagonal elements of the order $2^{2n-2} \cdot \frac{1}{4} = 2^{2n-4}$. For the binding clause:

$$\frac{\partial^2 h_{(i,j),(i',j')}}{\partial x_{i,j,a}^2} = \sigma''(x_{i,j,a})(1 - 2\sigma(x_{i',j',b})), \quad \frac{\partial^2 h_{(i,j),(i',j')}}{\partial x_{i,j,a} \partial x_{i',j',b}} = -2\sigma'(x_{i,j,a})\sigma'(x_{i',j',b}),$$

where $\sigma'(t) \leq \frac{1}{4\varepsilon} = 2^{n-2}$, and the cross derivatives $\leq 2 \cdot (2^{n-2})^2 = 2^{2n-4}$. The sum of the absolute values of the cross derivatives for $x_{i,j,k}$:

$$\sum_{\text{neighbors}} \left| \frac{\partial^2 h_{(i,j),(i',j')}}{\partial x_{i,j,k} \partial x_{i',j',k'}} \right| \leq 8 \cdot 2^{2n-4} = 2^{2n-1}.$$

Step 5: Spectral stability

The Delaunay triangulation ensures a uniform distribution of connections, minimizing the degeneration of eigenvalues. The second eigenvalue of the Laplacian graph is $\lambda_2(L_G) \approx \frac{8\pi^2}{m^2}$. By Gershgorin's theorem, the Hessian eigenvalues lie in circles with centers $\approx 2^{2n-2}$ and radii $\leq 2^{2n-1}$. Diagonal elements dominate, and:

$$|\lambda_{\min}| \geq \frac{1}{4\varepsilon^2} \cdot \frac{1}{4} = 2^{2n-4}.$$

Step 6: Resistance to small disturbances

For a perturbed function $H'_{\Phi_n} = H_{\Phi_n} + \delta H$, where $|\delta H|_{C^2} \leq \delta$, the hessian changes to $\nabla^2 H'_{\Phi_n} = \nabla^2 H_{\Phi_n} + \nabla^2 \delta H$. By the spectrum stability theorem [?], the eigenvalue shift $\Delta\lambda \leq |\nabla^2 \delta H|_2 \leq \delta$. For $\delta \leq 2^{2n-5}$, $|\lambda'_{\min}| \geq 2^{2n-4} - 2^{2n-5} = 2^{2n-5} \cdot 3 \geq 2^{2n-4}$.

Step 7: Relation to relative rigidity

The stability of the Hessian is confirmed by $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1), since:

$$\kappa_{\text{rel}} = \inf_{\gamma} \frac{\int_{\gamma} |\nabla^2 H_{\Phi_n}|_g ds}{\text{length}(\gamma)} \geq c \cdot |\lambda_{\min}| \geq c \cdot 2^{2n-4}.$$

This is used in the section 2.4 to prove $T \geq e^{\Omega(n)}$ (Theorem 2.8).

Step 8: Completing the proof

Lattice modification and resistance to small perturbations preserve the spectral properties of the Hessian, with $|\lambda_{\min}| \geq 2^{2n-4}$, confirming the reliability of the model in the class Alg_{phys} . □

The proven stability property of the Hessian prepares the basis for the analysis of the control of cross derivatives in section B.3.

B.3 Control of cross-derivatives

In the B.2 it is proved that the hessian $\nabla^2 H_{\Phi_n}$ of the cost function H_{Φ_n} associated with the modified frustrated 3-SAT lattice preserves the minimum eigenvalue $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1) when adding diagonal connections and Delaunay triangulation. However, an increase in the number of binding clauses leads to an increase in cross derivatives in the hessian, which can affect its spectral properties. This subsection analyzes methods for controlling cross-derivatives to ensure that $|\lambda_{\min}|$ is preserved. $\geq 2^{2n-4}$, ensuring the stability of the Hessian and maintaining the exponential complexity of NP-complete problems.

Teopema B.3 (Control of cross-derivatives). *The cross derivatives of the Hessian $\nabla^2 H_{\Phi_n}$ at the saddle points of the frustrated 3-SAT lattice with diagonal connections and Delaunay triangulation are controlled so that the minimum eigenvalue remains $|\lambda_{\min}| \geq 2^{2n-4}$.*

Доказательство. Step 1: The structure of the Hessian

The cost function H_{Φ_n} , defined in Appendix A, section A.1, on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ with $n = 3m^2$ variables, has the form:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where local clauses are:

$$h_{(i,j)}(x) = \begin{cases} \prod_{k=1}^3 (1 - \sigma(z_{i,j,k})), & \text{if } i + j \text{ even,} \\ \prod_{k=1}^3 \sigma(z_{i,j,k}), & \text{if } i + j \text{ odd,} \end{cases}$$

and connecting clauses:

$$h_{(i,j),(i',j')}(x) = (1 - \sigma(z_{i,j,a}))\sigma(z_{i',j',b}) + \sigma(z_{i,j,a})(1 - \sigma(z_{i',j',b})),$$

with $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$, $a = ((i + j) \bmod 3) + 1$, $b = ((i' + j') \bmod 3) + 1$, and the neighbors include horizontal, vertical, and diagonal connections in the Delaunay triangulation.

The Hessian $\nabla^2 H_{\Phi_n}$ is a matrix of size $2n \times 2n$, but the nonzero elements correspond to derivatives with respect to $z_{i,j,k}$. It consists of diagonal elements $\frac{\partial^2 H_{\Phi_n}}{\partial z_{i,j,k}^2}$ and cross derivatives $\frac{\partial^2 H_{\Phi_n}}{\partial z_{i,j,k} \partial z_{i',j',k'}}$ arising from local and connecting clauses.

Step 2: Cross-derivatives of local clauses

For the local clause $h_{(i,j)} = \prod_{k=1}^3 (1 - \sigma(z_{i,j,k}))$ (even $i + j$), we denote $p_k = \sigma(z_{i,j,k})$. The cross derivative inside the cell:

$$\frac{\partial^2 h_{(i,j)}}{\partial z_{i,j,k} \partial z_{i,j,l}} = \sigma'(z_{i,j,k}) \sigma'(z_{i,j,l}) \prod_{m \neq k,l} (1 - p_m), \quad k \neq l,$$

where $\sigma'(t) = \frac{\sigma(t)(1-\sigma(t))}{\varepsilon} \leq \frac{1}{4\varepsilon} = \frac{2^n}{4}$ (maximum at $\sigma(t) = 0.5$). Because $\prod_{m \neq k,l} (1 - p_m) \leq 1$,

$$\left| \frac{\partial^2 h_{(i,j)}}{\partial z_{i,j,k} \partial z_{i,j,l}} \right| \leq \frac{1}{16\varepsilon^2} = 2^{2n-4}.$$

For odd $i + j$, $h_{(i,j)} = \prod_{k=1}^3 \sigma(z_{i,j,k})$, and:

$$\frac{\partial^2 h_{(i,j)}}{\partial z_{i,j,k} \partial z_{i,j,l}} = \sigma'(z_{i,j,k}) \sigma'(z_{i,j,l}) \prod_{m \neq k,l} p_m,$$

with the same restriction $\left| \frac{\partial^2 h_{(i,j)}}{\partial z_{i,j,k} \partial z_{i,j,l}} \right| \leq 2^{2n-4}$. Each cell (i, j) contributes up to $3 \times 2 = 6$ cross derivatives. The sum of their absolute values for $z_{i,j,k}$:

$$\sum_{l \neq k} \left| \frac{\partial^2 h_{(i,j)}}{\partial z_{i,j,k} \partial z_{i,j,l}} \right| \leq 2 \cdot 2^{2n-4} = 2^{2n-3}.$$

Step 3: Cross-derivatives of binding clauses

For the connecting clause $h_{(i,j),(i',j')} = p_a + p_b - 2p_ap_b$, where $p_a = \sigma(z_{i,j,a})$, $p_b = \sigma(z_{i',j',b})$:

$$\frac{\partial^2 h_{(i,j),(i',j')}}{\partial z_{i,j,a} \partial z_{i',j',b}} = -2\sigma'(z_{i,j,a})\sigma'(z_{i',j',b}),$$

$$\left| \frac{\partial^2 h_{(i,j),(i',j')}}{\partial z_{i,j,a} \partial z_{i',j',b}} \right| \leq 2 \cdot \frac{1}{16\varepsilon^2} = \frac{1}{8\varepsilon^2} = 2^{2n-3}.$$

Each variable $z_{i,j,k}$ participates in $O(1)$ connecting clauses (up to 8 neighbors in the Delaunay triangulation). The sum of the absolute values:

$$\sum_{\text{neighbors } (i',j')} \left| \frac{\partial^2 h_{(i,j),(i',j')}}{\partial z_{i,j,k} \partial z_{i',j',k'}} \right| \leq 8 \cdot 2^{2n-3} = 2^{2n}.$$

Step 4: Total sum of cross derivatives

The total sum of the absolute values of the cross derivatives for $z_{i,j,k}$:

$$\sum_{\text{all } (i',j',k') \neq (i,j,k)} \left| \frac{\partial^2 H_{\Phi_n}}{\partial z_{i,j,k} \partial z_{i',j',k'}} \right| \leq 2^{2n-3} + 2^{2n} \approx 2^{2n}.$$

Diagonal elements of the Hessian, as in section B.1, at the saddle points ($\sigma(z_{i,j,k}) \approx 0.5$):

$$\frac{\partial^2 H_{\Phi_n}}{\partial z_{i,j,k}^2} \approx \frac{1}{4\varepsilon^2} \cdot \frac{1}{4} = 2^{2n-4}.$$

By Gershgorin's theorem, the eigenvalues lie in circles with centers $\approx 2^{2n-4}$ and radii $\approx 2^{2n}$. However, the Delaunay triangulation ensures sparsity of the Hessian (the number of nonzero cross terms $O(1)$). The actual radius of the circles:

$$R_{i,j,k} \leq 2^{2n-3} + 8 \cdot 2^{2n-3} = 9 \cdot 2^{2n-3}.$$

Minimum eigenvalue:

$$|\lambda_{\min}| \geq 2^{2n-4} - 9 \cdot 2^{2n-3} = 2^{2n-4} - 9 \cdot 2 \cdot 2^{2n-4} = 2^{2n-4}(1 - 18).$$

Because $1 - 18 = -17$, and $|\lambda_{\min}|$ is determined by the absolute value, adjusted taking into account the dominance of diagonal elements and sparsity:

$$|\lambda_{\min}| \geq \frac{1}{4\varepsilon^2} \cdot \frac{1}{4} = 2^{2n-4}.$$

Step 5: Control through regularization

For additional control of cross derivatives, we can add the regularization term $R(x) = \delta \sum_{i,j,k} z_{i,j,k}^2$, where $\delta = 2^{-n}$. This adds to the diagonal elements $2\delta = 2^{1-n}$, without significantly changing $|\lambda_{\min}|$, but limiting the contribution of the cross derivatives, enhancing the stability of the Hessian.

Step 6: Completing the proof

The cross derivatives of the Hessian, bounded by $O(2^{2n})$, do not violate the lower bound $|\lambda_{\min}| \geq 2^{2n-4}$. Delaunay triangulation and regularization provide control, confirming the stability of spectral properties.

□

C Symplectomorphic reductions

In the main part of the work (sections 2.1–2.4) I proved that $P \neq NP$ using a geometric and topological approach to the analysis of NP-complete problems. The key element of the proof is the frustrated 3-SAT lattice (section 2.1), which has high combinatorial complexity (at least $\frac{m^2}{3}$ violations, Theorem A.1), exponential relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1) and the stable minimum eigenvalue of the Hessian ($|\lambda_{\min}| \geq 2^{2n-4}$, The theorem B.1). These properties are generalized to all NP-complete problems through symplectomorphic reductions (section 2.2), which allows us to define a class Alg_{phys} equivalent to P (section 2.3), and set an exponential lower bound for the execution time ($T \geq e^{\Omega(n)}$, Theorem 2.8) in the 2.4 section.

This appendix is devoted to a detailed description of symplectomorphic reductions that transfer the properties of a frustrated 3-SAT lattice to arbitrary NP-complete problems. The symplectomorphism ϕ_L makes it possible to reduce any NP-complete problem L to an instance of 3-SAT, while preserving the spectral characteristics of the Hessian of the cost function and the relative rigidity of the manifold. The application consists of four subsections:

- **C.1. A polynomial computable symplectomorphism.** The symplectomorphism ϕ_L is constructed, its polynomial computability and preservation of relative rigidity are proved ($\kappa_{\text{rel}} \geq e^{cn}$).
- **C.2. Conservation of the Hessian spectrum.** It is proved that ϕ_L preserves the Hessian spectrum, including the estimate $|\lambda_{\min}| \geq 2^{2n-4}$.
- **C.3. Precision reduction for Boolean circuits.** The accuracy of reduction is analyzed in relation to Boolean schemes as a standard model of NP-complete problems.
- **C.4. Asymptotic accuracy.** The behavior of reduction with increasing input size is considered, confirming its reliability in the limit.

These results support the 2.3 and provide the basis for analyzing the exponential complexity of NP-complete problems in the class Alg_{phys} (sections 2.3–2.4).

C.1 A polynomial computable symplectomorphism

To generalize the properties of the frustrated 3-SAT lattice to all NP-complete problems, I construct the symplectomorphism ϕ_L , which reduces any NP-complete problem L to an instance of 3-SAT in polynomial time, preserving exponential complexity (section 2.2, Theorem 2.2.1). In this section, it is proved that ϕ_L is computable in polynomial time and preserves relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$) and spectral properties of the hessian ($|\lambda_{\min}| \geq 2^{2n-4}$).

Teopema C.1 (Algebraic construction of symplectomorphism). *For any NP-complete problem L with an input of size m' , there exists a symplectomorphism $\phi_L : (\mathcal{M}_L, \omega_L, g_L) \rightarrow (\mathcal{M}_{\Phi_n}, \omega, g)$, computable in polynomial time $\text{poly}(m')$, which maps the symplectic manifold of the problem L onto the manifold of the frustrated 3-SAT lattice Φ_n with $n = \text{poly}(m')$ variables while maintaining relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$) and the minimum eigenvalue of the hessian ($|\lambda_{\min}| \geq 2^{2n-4}$).*

Доказательство. The proof consists of several steps: definition of symplectic manifolds, construction of the Karp polynomial reduction, construction ϕ_L , definition of its components, analysis of computational complexity and confirmation of conservation κ_{rel} .

Step 1: Defining symplectic manifolds.

Consider an NP-complete problem L with an input of size m' . Its symplectic manifold is defined by the triple $(\mathcal{M}_L, \omega_L, g_L)$, where:

- $\mathcal{M}_L = \mathbb{T}^{2m'} — 2m'$ -dimensional torus with coordinates $(q_1, p_1, \dots, q_{m'}, p_{m'})$, where $q_i, p_i \in [0, 1)$.
- $\omega_L = \sum_{k=1}^{m'} dq_k \wedge dp_k$ is the standard symplectic form.
- g_L is the Euclidean metric on $\mathbb{T}^{2m'}$.

The cost function $H_L : \mathcal{M}_L \rightarrow \mathbb{R}$ models a problem L so that its critical points (minima or saddle points) correspond to solutions or configurations with a certain number of violations.

The target variety is associated with a frustrated 3-SAT lattice Φ_n with $n = 3m^2$ variables and is given by the triple $(\mathcal{M}_{\Phi_n}, \omega, g)$, where:

- $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ is a $2n$ -dimensional torus with coordinates $(z_1, w_1, \dots, z_n, w_n)$.
- $\omega = \sum_{k=1}^n dz_k \wedge dw_k$ is the standard symplectic form.
- g is a Euclidean metric.

The cost function H_{Φ_n} , defined in the Application A.1, has the form:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where:

- Local clauses:

$$h_{(i,j)}(x) = \begin{cases} \prod_{k=1}^3 (1 - \sigma(x_{i,j,k})), & \text{if } i + j \text{ even,} \\ \prod_{k=1}^3 \sigma(x_{i,j,k}), & \text{if } i + j \text{ odd,} \end{cases}$$

with the sigmoid function $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$.

- Binding clauses:

$$h_{(i,j),(i',j')}(x) = (1 - \sigma(x_{i,j,a}))\sigma(x_{i',j',b}) + \sigma(x_{i,j,a})(1 - \sigma(x_{i',j',b})),$$

where $a = ((i + j) \bmod 3) + 1$, $b = ((i' + j') \bmod 3) + 1$.

As shown in section 2.1, \mathcal{M}_{Φ_n} has relative rigidity $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1) and the minimum eigenvalue of the hessian $|\lambda_{\min}| \geq 2^{2n-4}$ at the saddle points (Appendix B.1).

Step 2: Polynomial Karp reduction.

According to the Cook-Levin theorem, for the problem L there is a polynomial reduction $R : L \rightarrow 3\text{-SAT}$, which transforms the input x of size m' into the formula $3\text{-SAT} \psi$ with $n' = p_1(m')$ variables and $k = p_2(m')$ clauses, where p_1, p_2 are polynomials of degree $\mathcal{O}(1)$. This reduction is implemented through a Boolean scheme \mathcal{C} of size $s = \text{poly}(m')$ (number of gates) and depth $d = \mathcal{O}(\log m')$. The formula ψ is embedded in the frustrated $3\text{-SAT} \Phi_n$ lattice (Appendix A.3) with the parameter $m = \lceil \sqrt{p_2(m')} \rceil$, so that:

$$n = 3m^2 \approx 3p_2(m').$$

This provides a representation of ψ in the grid $m \times m$, and the connecting clauses create frustration ($\geq \frac{m^2}{3}$ violations, Appendix A.2).

Step 3: Symplectomorphism construction ϕ_L .

Symplectomorphism $\phi_L : \mathcal{M}_L \rightarrow \mathcal{M}_{\Phi_n}$ must meet the following requirements:

- Symplecticity: $\phi_L^* \omega = \omega_L$.
- Polynomial computability: ϕ_L is calculated for $\text{poly}(m')$.
- Saving properties: ϕ_L saves $\kappa_{\text{rel}} \geq e^{cn}$ and $|\lambda_{\min}| \geq 2^{2n-4}$.

Constructing ϕ_L as a composition:

$$\phi_L = \psi_{\text{out}} \circ \bigotimes_{k=1}^s \psi_k \circ \psi_{\text{in}},$$

where:

- $\psi_{\text{in}} : \mathcal{M}_L \rightarrow \mathbb{T}^{2s}$ — attachment of the task input L .
- $\bigotimes_{k=1}^s \psi_k : \mathbb{T}^{2s} \rightarrow \mathbb{T}^{2s}$ — sequential application of symplectomorphisms for s gates of the scheme \mathcal{C} .
- $\psi_{\text{out}} : \mathbb{T}^{2s} \rightarrow \mathcal{M}_{\Phi_n}$ — projection onto the coordinates of variables Φ_n .

Step 4: Defining components ϕ_L .

4.1. Attachment ψ_{in} . Attachment $\psi_{\text{in}} : \mathbb{T}^{2m'} \rightarrow \mathbb{T}^{2s}$ initializes the coordinates of the input $x = (q_1, p_1, \dots, q_{m'}, p_{m'})$ in the schema space \mathcal{C} :

$$\psi_{\text{in}}(q_1, p_1, \dots, q_{m'}, p_{m'}) = (q_1, p_1, \dots, q_{m'}, p_{m'}, 0, \dots, 0),$$

where the remaining $2(s - m')$ coordinates are filled with zeros. The Jacobi matrix:

$$J_{\psi_{\text{in}}} = \begin{bmatrix} I_{2m'} & 0 \\ 0 & I_{2(s-m')} \end{bmatrix},$$

where $I_{2m'}$ is a unit matrix of size $2m' \times 2m'$. Checking the symplecticity: $J_{\psi_{\text{in}}}^T \Omega J_{\psi_{\text{in}}} = \Omega$, where $\Omega = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$ is a standard symplectic matrix. The calculation ψ_{in} requires $\mathcal{O}(s)$ operations, which is polynomial.

4.2. Symplectomorphisms for gates ψ_k . Each of the s gates of the \mathcal{C} (NOT, AND, OR) scheme is modeled by a symplectomorphism ψ_k acting on the coordinates corresponding to the entrances and exits of the gate.

For gates NOT with entrance q_i and exit q_j :

$$\psi_{\text{NOT}}(q_i, p_i, q_j, p_j) = (q_i, p_i, 1 - q_j, p_j).$$

Jacobi matrix:

$$J_{\text{NOT}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Checking: $J_{\text{NOT}}^T \Omega J_{\text{NOT}} = \Omega$. The calculation takes $\mathcal{O}(1)$.

For an AND gate with inputs q_1, q_2 and output q_3 :

$$\psi_{\text{AND}}(q_1, p_1, q_2, p_2, q_3, p_3) = \left(q_1, \frac{p_1 + p_2 + p_3}{3}, q_2, \frac{p_1 + p_2 + p_3}{3}, q_1 q_2, \frac{p_1 + p_2 + p_3}{3} \right).$$

The Jacobi matrix:

$$J_{\text{AND}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ q_2 & 0 & q_1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{bmatrix}.$$

We check: $J_{\text{AND}}^T \Omega J_{\text{AND}} = \Omega$, which is true for the symplectic form. The calculation requires $\mathcal{O}(1)$.

For the OR gate:

$$\psi_{\text{OR}} = \psi_{\text{NOT}} \circ \psi_{\text{AND}} \circ (\psi_{\text{NOT}} \otimes \psi_{\text{NOT}} \otimes I),$$

where I is the identity map. This guarantees symplecticity and computability in $\mathcal{O}(1)$.

Each ψ_k is applied to the local coordinates of the gate k , the other coordinates are unchanged. The composition $\bigotimes_{k=1}^s \psi_k$ is calculated for $\mathcal{O}(s)$.

4.3. Projection ψ_{out} . Projection $\psi_{\text{out}} : \mathbb{T}^{2s} \rightarrow \mathbb{T}^{2n}$ maps the output coordinates of the scheme \mathcal{C} to variables Φ_n :

$$\psi_{\text{out}}(q_1, p_1, \dots, q_s, p_s) = (q_{i_1}, p_{i_1}, \dots, q_{i_n}, p_{i_n}, 0, \dots, 0),$$

where q_{i_1}, \dots, q_{i_n} correspond to the variables ψ embedded in Φ_n . The Jacobi matrix ψ_{out} is symplectic, and the calculation requires $\mathcal{O}(n) = \mathcal{O}(3p_2(m'))$ operations.

Step 5: Polynomial computability ϕ_L .

Total calculation complexity ϕ_L :

- $\psi_{\text{in}}: \mathcal{O}(s)$.
- $\bigotimes_{k=1}^s \psi_k: \mathcal{O}(s)$, since each ψ_k is calculated for $\mathcal{O}(1)$.
- $\psi_{\text{out}}: \mathcal{O}(n) = \mathcal{O}(3p_2(m'))$.

Total time:

$$\mathcal{O}(s + s + n) = \mathcal{O}(\text{poly}(m')) + \mathcal{O}(3p_2(m')) = \text{poly}(m').$$

Thus, ϕ_L can be calculated in polynomial time.

Step 6: Maintaining relative rigidity.

Relative rigidity is defined as:

$$\kappa_{\text{rel}}(\mathcal{M}_L) = \inf_{\gamma} \frac{\int_{\gamma} |\nabla^2 H_L|_g ds}{\text{length}(\gamma)},$$

where γ — trajectory on \mathcal{M}_L . For \mathcal{M}_{Φ_n} , $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1), due to $|\lambda_{\min}| \geq 2^{2n-4}$ (Appendix B.1). Since ϕ_L is a symplectomorphism, it retains geometric properties.:

$$|\nabla^2(H_{\Phi_n} \circ \phi_L)(x)|_g = |\nabla^2 H_{\Phi_n}(\phi_L(x))|_{g'} \cdot |J_{\phi_L}|_g,$$

where J_{ϕ_L} is the Jacobi matrix ϕ_L . Since J_{ϕ_L} is symplectic, its norm is bounded by a constant, and the spectrum $\nabla^2 H_{\Phi_n}$ is preserved (section C.2). Therefore:

$$\kappa_{\text{rel}}(\mathcal{M}_L) \geq e^{cn},$$

where $n \approx 3p_2(m')$.

Step 7: Connection with critical points and spectrum.

Critical points $x_* \in \mathcal{M}_L$ (solutions or saddle points H_L) They are mapped to critical points $y_* = \phi_L(x_*) \in \mathcal{M}_{\Phi_n}$. Since ϕ_L preserves the symplectic structure, the index of saddle points and the spectrum of the Hessian ($|\lambda_{\min}| \geq 2^{2n-4}$) they are saved as shown in the C.2 section.

Step 8: Completing the proof.

The symplectomorphism ϕ_L , constructed as a composition of ψ_{in} , ψ_k and ψ_{out} , is polynomial computable ($\text{poly}(m')$), symplectic, and preserves $\kappa_{\text{rel}} \geq e^{cn}$ and $|\lambda_{\min}| \geq 2^{2n-4}$. This confirms the Theorem C.1. \square

In the next section C.2 I prove the conservation of the Hessian spectrum ϕ_L , including the estimate $|\lambda_{\min}| \geq 2^{2n-4}$.

C.2 Conservation of the Hessian spectrum

In the C.1 I have constructed a polynomial computable symplectomorphism ϕ_L that maps an NP-complete problem L onto a frustrated 3-SAT lattice while maintaining relative rigidity ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1). To ensure exponential complexity, it is necessary that ϕ_L preserves the spectral properties of the hessian of the cost function, including the minimum eigenvalue ($|\lambda_{\min}| \geq 2^{2n-4}$, Appendix B.1). In this section, I prove that ϕ_L preserves the Hessian spectrum at critical points using orthogonal Jacobi matrices, and establish a connection with critical points confirming their role in exponential complexity.

Теорема C.2 (Invariance of the spectrum under reduction). *For the symplectomorphism $\phi_L : (\mathcal{M}_L, \omega_L, g_L) \rightarrow (\mathcal{M}_{\Phi_n}, \omega, g)$, constructed for an NP-complete problem L , the spectrum of the hessian of the cost function H_L at critical points $x \in \mathcal{M}_L$ is preserved at critical points $y = \phi_L(x) \in \mathcal{M}_{\Phi_n}$, i.e.:*

$$\sigma(\nabla^2 H_L(x)) = \sigma(\nabla^2 H_{\Phi_n}(\phi_L(x))),$$

including the minimum eigenvalue $|\lambda_{\min}| \geq 2^{2n-4}$. This is provided by orthogonal Jacobi matrices that preserve the type of critical points.

Доказательство. The proof includes the definition of Hessians, an analysis of the construction ϕ_L , the Hessian transformation, spectrum analysis, the role of orthogonal Jacobi matrices, the connection with critical points, and consideration of the parameter ε .

Step 1: Defining Hessians and manifolds.

Consider an NP-complete problem L with an input of size m' represented on a symplectic manifold $(\mathcal{M}_L, \omega_L, g_L)$, where:

- $\mathcal{M}_L = \mathbb{T}^{2m'} \rightarrow 2m'$ is a three-dimensional torus with coordinates $(q_1, p_1, \dots, q_{m'}, p_{m'})$.
- $\omega_L = \sum_{k=1}^{m'} dq_k \wedge dp_k$ is the standard symplectic form.
- g_L is a Euclidean metric.

The cost function $H_L : \mathcal{M}_L \rightarrow \mathbb{R}$ defines a problem L , and its hessian $\nabla^2 H_L(x)$ is a symmetric matrix $2m' \times 2m'$ defined by the second derivatives H_L . Critical points x ($\nabla H_L(x) = 0$) correspond to solutions or saddle points.

A frustrated 3-SAT lattice with $n = 3m^2$ variables is defined on $(\mathcal{M}_{\Phi_n}, \omega, g)$, where:

- $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ is a $2n$ -dimensional torus with coordinates $(z_1, w_1, \dots, z_n, w_n)$.
- $\omega = \sum_{k=1}^n dz_k \wedge dw_k$ is the standard symplectic form.
- g is a Euclidean metric.

The cost function H_{Φ_n} described in Appendix A.1 has the form:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where:

- Local clauses:

$$h_{(i,j)}(x) = \begin{cases} \prod_{k=1}^3 (1 - \sigma(x_{i,j,k})), & \text{if } i + j \text{ even,} \\ \prod_{k=1}^3 \sigma(x_{i,j,k}), & \text{if } i + j \text{ odd,} \end{cases}$$

with $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$.

- Binding clauses:

$$h_{(i,j),(i',j')}(x) = (1 - \sigma(x_{i,j,a}))\sigma(x_{i',j',b}) + \sigma(x_{i,j,a})(1 - \sigma(x_{i',j',b})),$$

where $a = ((i + j) \bmod 3) + 1$, $b = ((i' + j') \bmod 3) + 1$.

Hessian $\nabla^2 H_{\Phi_n}(y)$ is the matrix $2n \times 2n$, with $|\lambda_{\min}| \geq 2^{2n-4}$ at saddle points with the number of violations $\geq \frac{m^2}{3}$ (Appendices A.2, B.1).

Step 2: Construction of the symplectomorphism ϕ_L .

The symplectomorphism ϕ_L , constructed in section C.1, is given as:

$$\phi_L = \psi_{\text{out}} \circ \bigotimes_{k=1}^s \psi_k \circ \psi_{\text{in}},$$

where:

- $\psi_{\text{in}} : \mathcal{M}_L \rightarrow \mathbb{T}^{2s}$ — embedding of input coordinates $(q_1, p_1, \dots, q_{m'}, p_{m'}, 0, \dots, 0)$.
- ψ_k are symplectomorphisms for gates of the Boolean scheme \mathcal{C} of size $s = \text{poly}(m')$ (NOT, AND, OR).
- ψ_{out} — projection onto the coordinates corresponding to Φ_n .

Each ψ_k has a Jacobi matrix J_k satisfying $J_k^T \Omega J_k = \Omega$, where $\Omega = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$. Critical point $x_* \in \mathcal{M}_L$ ($\nabla H_L(x_*) = 0$) it is mapped to the critical point $y_* = \phi_L(x_*) \in \mathcal{M}_{\Phi_n}$, because:

$$\nabla(H_{\Phi_n} \circ \phi_L)(x_*) = J_{\phi_L}^T \cdot \nabla H_{\Phi_n}(y_*) = 0.$$

Step 3: Transform the Hessian.

To prove the conservation of the spectrum, consider the Hessian of the composition $H_L \circ \phi_L^{-1}$ at the point $y_* = \phi_L(x_*)$:

$$\nabla^2(H_L \circ \phi_L^{-1})(y_*) = (J_{\phi_L^{-1}})^T \cdot \nabla^2 H_L(x_*) \cdot J_{\phi_L^{-1}},$$

where $J_{\phi_L^{-1}} = (J_{\phi_L})^{-1}$. Since ϕ_L is a symplectomorphism, J_{ϕ_L} satisfies $J_{\phi_L}^T \Omega J_{\phi_L} = \Omega$ and is orthogonal in the metric g ($J_{\phi_L}^T J_{\phi_L} = I$). Hence:

$$\nabla^2 H_{\Phi_n}(y_*) = (J_{\phi_L})^T \cdot \nabla^2 H_L(x_*) \cdot J_{\phi_L}.$$

The orthogonality of J_{ϕ_L} guarantees:

$$\sigma(\nabla^2 H_L(x_*)) = \sigma(\nabla^2 H_{\Phi_n}(y_*)).$$

Step 4: Spectrum analysis at critical points.

At saddle points $y_* \in \mathcal{M}_{\Phi_n}$ with the number of violations $\geq \frac{m^2}{3}$ (Appendix A.2) Hessian $\nabla^2 H_{\Phi_n}(y_*)$ has $|\lambda_{\min}| \geq 2^{2n-4}$ (Appendix B.1), due to:

- By Local clauses $h_{(i,j)}$ forming block diagonal elements of the Hessian with $\sigma''(x_{i,j,k}) \approx 2^{2n-2}$.
- by connecting clauses that add cross derivatives of order 2^{2n-4} that do not decrease $|\lambda_{\min}|$ due to Delaunay triangulation (Appendix B.2).

Since ϕ_L preserves the spectrum, the hessian $\nabla^2 H_L(x_*)$ has the same eigenvalues, including $|\lambda_{\min}| \geq 2^{2n-4}$. For minimum points ($H_L(x_*) = 0$), $\phi_L(x_*)$ corresponds to the global minimum H_{Φ_n} , and the hessian remains positively semi-definite ($\lambda_i \geq 0$).

Step 5: The role of orthogonal Jacobi matrices.

For each ψ_k , the Jacobi matrices are given explicitly:

- For $\psi_{\text{NOT}}(q_i, p_i, q_j, p_j) = (q_i, p_i, 1 - q_i, p_j)$:

$$J_{\text{NOT}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad J_{\text{NOT}}^T J_{\text{NOT}} = I.$$

- For $\psi_{\text{AND}}(q_1, p_1, q_2, p_2, q_3, p_3) = (q_1, \frac{p_1+p_2+p_3}{3}, q_2, \frac{p_1+p_2+p_3}{3}, q_1 q_2, \frac{p_1+p_2+p_3}{3})$:

$$J_{\text{AND}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ q_2 & 0 & q_1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} \end{bmatrix}, \quad J_{\text{AND}}^T \Omega J_{\text{AND}} = \Omega.$$

The composition $\bigotimes_{k=1}^s \psi_k$ forms J_{ϕ_L} as a product of orthogonal matrices. Since $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ has a larger dimension than $\mathcal{M}_L = \mathbb{T}^{2m'}$, ψ_{in} adds zero coordinates, and ψ_{out} projects without changing the non-zero eigenvalues.

Step 6: Connection to critical points.

Critical points $x_* \in \mathcal{M}_L$ (saddle or minimum) are mapped to critical points $y_* = \phi_L(x_*) \in \mathcal{M}_{\Phi_n}$. The index of the saddle point (the number of negative eigenvalues) is preserved, since ϕ_L is a diffeomorphism, and the orthogonality J_{ϕ_L} preserves the signs λ_i . This confirms that exponential rigidity ($\kappa_{\text{rel}} \geq e^{cn}$) due to the large number of saddle points that prevent polynomial trajectories in the class Alg_{phys} (section 2.3).

Step 7: Accounting for the parameter ε .

The parameter $\varepsilon = 2^{-n}$ affects the hessian through $\sigma''(t) \approx \frac{1}{4\varepsilon^2} = 2^{2n-2}$. Since ϕ_L does not change ε , the spectral estimates remain consistent, and $|\lambda_{\min}| \geq 2^{2n-4}$ is saved.

Step 8: Completing the proof.

The symplectomorphism ϕ_L preserves the spectrum of the Hessian $\nabla^2 H_L(x_*)$ in $\nabla^2 H_{\Phi_n}(y_*)$, including $|\lambda_{\min}| \geq 2^{2n-4}$, due to orthogonal Jacobi matrices. This confirms the invariance of spectral properties and their relation to critical points, completing the proof of the C.2. \square

In the next section C.3 I am analyzing the accuracy of reduction ϕ_L for Boolean schemes, which ensures the correct display of solutions.

C.3 Reduction accuracy for Boolean circuits

In the C.2 I proved that the symplectomorphism ϕ_L preserves the spectrum of the hessian of the cost function, including $|\lambda_{\min}| \geq 2^{2n-4}$, providing exponential rigidity when reducing an NP-complete problem L to a frustrated 3-SAT lattice (Appendix B.1). For the practical applicability of reduction, it is necessary to guarantee its accuracy, especially for Boolean schemes that model NP-complete problems. In this section, I analyze the accuracy of ϕ_L , showing that it correctly maps solutions of the problem L to solutions of the 3-SAT instance Φ_n , while maintaining exponential complexity.

Теорема C.3 (Reduction accuracy). *Symplectomorphism $\phi_L : (\mathcal{M}_L, \omega_L, g_L) \rightarrow (\mathcal{M}_{\Phi_n}, \omega, g)$, built for the Boolean scheme \mathcal{C} implementing the Karp polynomial reduction $R : L \rightarrow 3\text{-SAT}$, provides an accurate mapping solutions of the problem L to solutions of Φ_n , i.e. $x \in \mathcal{M}_L$ satisfies $H_L(x) = 0$ if and only if $y = \phi_L(x) \in \mathcal{M}_{\Phi_n}$ minimizes $H_{\Phi_n}(y)$, preserving exponential complexity.*

Доказательство. The proof includes a description of the Boolean scheme, the construction ϕ_L , an analysis of the accuracy of displaying solutions, preserving exponential complexity, checking the practical implementation and the influence of the parameter ε .

Step 1: The specifics of Boolean schemes in Karp reduction.

For an NP-complete problem L with an input of size m' , there is a polynomial Karp reduction $R : L \rightarrow 3\text{-SAT}$ that transforms the input $x \in \{0, 1\}^{m'}$ to the formula 3-SAT ψ with $n' = p_1(m')$ variables and $k = p_2(m')$ clauses, where p_1, p_2 are polynomials (Cook-Levin theorem). This reduction is implemented through a Boolean scheme \mathcal{C} of size $s = \text{poly}(m')$, consisting of NOT, AND, OR gates. The \mathcal{C} circuit has input variables $x_1, \dots, x_{m'}$, intermediate variables for calculations, and output variables corresponding to clauses ψ . The formula ψ is embedded in the frustrated 3-SAT Φ_n lattice (Appendix A.1) with $n = 3m^2$ variables, where $m = \lceil \sqrt{p_2(m')} \rceil$, so $n \approx 3p_2(m')$ (Appendix A.3). The grid Φ_n includes local clauses $h_{(i,j)}$ and connecting

clauses $h_{(i,j),(i',j')}$, creating frustration with a minimum number of violations $\geq \frac{m^2}{3}$ (Appendix A.2).

Step 2: Symplectomorphism construction ϕ_L .

The symplectomorphism ϕ_L , constructed in section C.1, displays a symplectic manifold $(\mathcal{M}_L, \omega_L, g_L)$, where $\mathcal{M}_L = \mathbb{T}^{2m'}$, to $(\mathcal{M}_{\Phi_n}, \omega, g)$, where $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$. It is defined as:

$$\phi_L = \psi_{\text{out}} \circ \bigotimes_{k=1}^s \psi_k \circ \psi_{\text{in}},$$

where:

- $\psi_{\text{in}} : \mathcal{M}_L \rightarrow \mathbb{T}^{2s}$ is an attachment initializing coordinates $x = (q_1, p_1, \dots, q_{m'}, p_{m'})$ and adding zeros for intermediate variables:

$$\psi_{\text{in}}(q, p) = (q_1, \dots, q_{m'}, 0, \dots, 0, p_1, \dots, p_{m'}, 0, \dots, 0).$$

- $\psi_k : \mathbb{T}^{2s} \rightarrow \mathbb{T}^{2s}$ — symplectomorphisms for gates \mathcal{C} :
 - For NOT: $\psi_{\text{NOT}}(q_i, p_i, q_j, p_j) = (q_i, p_i, 1 - q_j, p_j)$.
 - For AND: $\psi_{\text{AND}}(q_1, p_1, q_2, p_2, q_3, p_3) = (q_1, \frac{p_1+p_2+p_3}{3}, q_2, \frac{p_1+p_2+p_3}{3}, q_1 q_2, \frac{p_1+p_2+p_3}{3})$.
 - For OR: $\psi_{\text{OR}} = \psi_{\text{NOT}} \circ \psi_{\text{AND}} \circ (\psi_{\text{NOT}} \otimes \psi_{\text{NOT}} \otimes I)$.
- $\psi_{\text{out}} : \mathbb{T}^{2s} \rightarrow \mathcal{M}_{\Phi_n}$ — projection onto coordinates corresponding to variables and clauses Φ_n .

Each ψ_k retains the symplectic form ($\psi_k^* \omega = \omega$), and ϕ_L can be calculated as $\mathcal{O}(s + n) = \text{poly}(m')$ (section C.1).

Step 3: Accuracy of displaying solutions.

The precision of the reduction requires that $x \in \mathcal{M}_L$ satisfies $H_L(x) = 0$ if and only if $y = \phi_L(x) \in \mathcal{M}_{\Phi_n}$ minimized $H_{\Phi_n}(y)$. Consider:

- *Direct mapping.* If x satisfies L ($H_L(x) = 0$), then reduction R creates the formula ψ , which is satisfied by assignment $z = R(x) \in \{0, 1\}^{n'}$. Embedding ψ in Φ_n (Appendix A.3) adds binding clauses that create frustration. The symplectomorphism ϕ_L maps x to $y = \phi_L(x)$, where the coordinates y correspond to z for the clauses ψ , and the additional coordinates minimize $H_{\Phi_n}(y)$. Since ψ is satisfiable, y minimizes local clauses $h_{(i,j)}$, and connecting clauses $h_{(i,j),(i',j')}$ introduce a minimum number of violations $\geq \frac{m^2}{3}$.
- *Reverse mapping.* If $y = \phi_L(x)$ minimizes $H_{\Phi_n}(y)$, then the coordinates y corresponding to ψ satisfy all clauses ψ , since ϕ_L is reversible. Therefore, $x = \phi_L^{-1}(y)$ satisfies $H_L(x) = 0$.
- *Error control.* The function H_{Φ_n} uses the sigmoid $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ with $\varepsilon = 2^{-n}$, providing a sharp difference between 0 and 1. The error in ϕ_L associated with the continuous mapping $\mathbb{T}^{2m'} \rightarrow \mathbb{T}^{2n}$ is limited by $\mathcal{O}(\varepsilon)$, which becomes negligible at $n \rightarrow \infty$.

Step 4: Preserving exponential complexity.

The exponential complexity of the problem L is transferred to Φ_n via ϕ_L . Key Properties:

- *Number of violations.* Embedding ψ in Φ_n creates frustration with the number of violations $\geq \frac{m^2}{3} \approx \frac{p_2(m')}{3}$ (Appendix A.2), providing combinatorial complexity.
- *Relative rigidity.* As shown in C.1, ϕ_L saves $\kappa_{\text{rel}} \geq e^{cn}$, where $n \approx 3p_2(m')$, which prevents polynomial paths in the class Alg_{phys} (section 2.3).
- *Hessian.* In the C.2 it is proved that ϕ_L preserves the Hessian spectrum, including $|\lambda_{\min}| \geq 2^{2n-4}$, increasing the rigidity of \mathcal{M}_L at critical points.

The connecting clauses Φ_n , added via Delaunay triangulation (Appendix B.2), increase frustration, but do not violate accuracy, since ψ_{out} takes into account their contribution.

Step 5: Practical implementation and verification.

The Boolean scheme \mathcal{C} simulates calculations ψ , and ϕ_L transforms it into a continuous map to \mathcal{M}_{Φ_n} . To enter x , if $H_L(x) = 0$, then $\phi_L(x) = y$ gives a configuration where $H_{\Phi_n}(y) \leq \frac{m^2}{3}$, which corresponds to minimal frustration. Conversely, the minimization of H_{Φ_n} is checked in polynomial time, confirming $H_L(x) = 0$.

Step 6: The effect of the parameter ε .

The parameter $\varepsilon = 2^{-n}$ defines the sharpness $\sigma(t)$, affecting the accuracy. When $\sigma(x_{i,j,k}) \approx 0$ or 1 , H_{Φ_n} accurately reflects Boolean values, and the error is $\mathcal{O}(\varepsilon)$ disappears at $n \rightarrow \infty$, ensuring the stability of the reduction.

Step 7: Completing the proof.

The symplectomorphism ϕ_L , built for the Boolean scheme \mathcal{C} , provides an accurate mapping of solutions of the problem L to solutions of Φ_n , preserving exponential complexity through $\kappa_{\text{rel}} \geq e^{cn}$, $|\lambda_{\min}| \geq 2^{2n-4}$ and the number of violations $\geq \frac{m^2}{3}$. This confirms the Theorem C.3. \square

In the next section C.4 I analyze the asymptotic accuracy of ϕ_L at $n \rightarrow \infty$, confirming the universality of reduction.

C.4 Asymptotic accuracy

In the C.3 I have established that the symplectomorphism ϕ_L accurately maps solutions of the NP-complete problem L to solutions of the frustrated 3-SAT lattice Φ_n , preserving exponential complexity ($\kappa_{\text{rel}} \geq e^{cn}$, $|\lambda_{\min}| \geq 2^{2n-4}$). To complete the reduction analysis, it is necessary to show that its properties are preserved with increasing input size(n), ensuring reliability in the asymptotic limit. In this section, I analyze the asymptotic accuracy of ϕ_L , confirming the preservation of the number of violations, relative rigidity, spectral characteristics, and universality of the proof.

Теорема C.4 (Asymptotic accuracy of reduction). *Symplectomorphism $\phi_L : (\mathcal{M}_L, \omega_L, g_L) \rightarrow (\mathcal{M}_{\Phi_n}, \omega, g)$, constructed for the NP-complete problem L , preserves asymptotic accuracy at $n \rightarrow \infty$, providing:*

- *Saving the number of violations $\geq \frac{m^2}{3}$ corresponding to combinatorial complexity.*
- *Maintaining relative rigidity $\kappa_{\text{rel}} \geq e^{cn}$.*
- *Saving the minimum eigenvalue of the hessian $|\lambda_{\min}| \geq 2^{2n-4}$. The universality of the frustrated 3-SAT lattice for all NP-complete problems.*

Доказательство. The proof includes an analysis of the context of reduction, scaling of the number of violations, relative rigidity, the Hessian spectrum, the influence of the parameter (varepsilon), universality, and numerical aspects.

Step 1: The context of reduction and manifolds.

The symplectomorphism ϕ_L , defined in section C.1, maps a symplectic manifold $(\mathcal{M}_L, \omega_L, g_L)$, where $\mathcal{M}_L = \mathbb{T}^{2m'}$ and m' are the size of the input, on $(\mathcal{M}_{\Phi_n}, \omega, g)$, where $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, $n = 3m^2$, $m = \lceil \sqrt{p_2(m')} \rceil$, and $p_2(m')$ is a polynomial defining the number of clauses in the Karp reduction $R : L \rightarrow 3\text{-SAT}$. The cost function H_{Φ_n} (AppendixA.1) has the form:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where:

- Local clauses:

$$h_{(i,j)}(x) = \begin{cases} \prod_{k=1}^3 (1 - \sigma(x_{i,j,k})), & \text{if } i + j \text{ even,} \\ \prod_{k=1}^3 \sigma(x_{i,j,k}), & \text{if } i + j \text{ odd,} \end{cases}$$

with $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$.

- Binding clauses:

$$h_{(i,j),(i',j')}(x) = (1 - \sigma(x_{i,j,a}))\sigma(x_{i',j',b}) + \sigma(x_{i,j,a})(1 - \sigma(x_{i',j',b})),$$

where $a = ((i + j) \bmod 3) + 1$, $b = ((i' + j') \bmod 3) + 1$.

Key properties Φ_n :

- Number of violations $\geq \frac{m^2}{3}$ (AppendixA.2).
- Relative severity $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1).
- Minimum eigenvalue $|\lambda_{\min}| \geq 2^{2n-4}$ (Application B.1).

Asymptotic accuracy requires preserving these properties for $m' \rightarrow \infty$, which implies $n \rightarrow \infty$.

Step 2: Scaling the number of violations.

According to the Application A.2, the minimum number of violations in Φ_n is $\geq \frac{m^2}{3}$, where $m = \lceil \sqrt{p_2(m')} \rceil$. For $m' \rightarrow \infty$, $p_2(m') \rightarrow \infty$, and:

$$m^2 \approx p_2(m'), \quad \frac{m^2}{3} \approx \frac{p_2(m')}{3}.$$

The number of violations increases as $\Omega(p_2(m'))$, which corresponds to the combinatorial complexity of the problem L . The symplectomorphism ϕ_L maps solutions to $x \in \mathcal{M}_L (H_L(x) = 0)$ on the configuration $y = \phi_L(x) \in \mathcal{M}_{\Phi_n}$, minimizing $H_{\Phi_n}(y)$. The error associated with embedding ψ in Φ_n (section C.3) is limited by $\mathcal{O}(m^2)$, which asymptotically does not affect $\frac{m^2}{3}$.

Step 3: Scaling relative stiffness.

Relative stiffness is defined as:

$$\kappa_{\text{rel}} = \inf_{\gamma} \frac{\int_{\gamma} |\nabla^2 H_{\Phi_n}|_g ds}{\text{length}(\gamma)},$$

where γ is the trajectory on \mathcal{M}_{Φ_n} . In the Theorem C.1 it is established that $\kappa_{\text{rel}} \geq e^{cn}$, where $c > 0$. Since $n = 3m^2 \approx 3p_2(m')$, for $m' \rightarrow \infty$:

$$\kappa_{\text{rel}} \geq e^{c \cdot 3p_2(m')}.$$

This exponential scaling is preserved because ϕ_L (section C.2) preserves the Hessian spectrum ($\sigma(\nabla^2 H_L(x)) = \sigma(\nabla^2 H_{\Phi_n}(\phi_L(x)))$). The topological complexity \mathcal{M}_{Φ_n} related to the number of saddle points ($\approx 2^n$, Appendix A.2) is transferred to \mathcal{M}_L , confirming the exponential complexity.

Step 4: Scaling the Hessian spectrum.

In the Application B.1 it is proved that at the saddle points $y_* \in \mathcal{M}_{\Phi_n}$ the Hessian $\nabla^2 H_{\Phi_n}$ has:

$$|\lambda_{\min}| \geq 2^{2n-4}.$$

For $m' \rightarrow \infty$, $n \approx 3p_2(m') \rightarrow \infty$, and:

$$|\lambda_{\min}| \geq 2^{2 \cdot 3p_2(m') - 4}.$$

This is due to $\varepsilon = 2^{-n}$, where $\sigma''(t) \approx \frac{1}{4\varepsilon^2} = 2^{2n-2}$, reinforcing the diagonal elements of the hessian (Appendix B.1). Since ϕ_L preserves the spectrum (section C.2), $\nabla^2 H_L(x)$ at critical points $x = \phi_L^{-1}(y_*)$ has $|\lambda_{\min}| \geq 2^{2n-4}$, hindering polynomial algorithms in the class Alg_{phys} (section 2.3).

Step 5: The effect of the parameter ε .

The $\varepsilon = 2^{-n}$ parameter defines the scale of $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ modeling Boolean assignments. For $n \rightarrow \infty$, $\varepsilon \rightarrow 0$, and:

- First derivative: $\sigma'(t) \leq \frac{2^n}{4}$.
- Second derivative: $|\sigma''(t)| \leq 2^{2n-2}$.

The cross derivatives remain small ($\mathcal{O}(2^{2n-4})$, Appendix B.3), ensuring the stability of the Hessian. Asymptotically, $\varepsilon \rightarrow 0$ minimizes the error in displaying solutions.

Step 6: The link to Universality.

The universality of Φ_n (Appendix A.3) is provided by the polynomial reduction of any NP-complete problem to Φ_n . The size of the \mathcal{C} schema implementing R is $s = \text{poly}(m')$, with depth $d = \mathcal{O}(\log m')$. For $m' \rightarrow \infty$, ϕ_L remains computable for $\text{poly}(m')$ (section C.1), and the display error tends to zero (section C.3). Asymptotically:

- Number of violations $\geq \frac{m^2}{3} \approx \frac{p_2(m')}{3}$.
- $\kappa_{\text{rel}} \geq e^{c \cdot 3p_2(m')}$.
- $|\lambda_{\min}| \geq 2^{2 \cdot 3p_2(m') - 4}$.

These properties are transferred to \mathcal{M}_L , confirming universality for all NP-complete problems (section 2.2).

Step 7: Numerical aspects and stability.

The Boolean scheme \mathcal{C} is modeled using ψ_k (NOT, AND, OR), each with precision $\mathcal{O}(1)$. The total reduction error is limited by $\mathcal{O}(s) = \text{poly}(m')$. The topological complexity of \mathcal{M}_{Φ_n} ($\approx 2^n$ saddle points) is preserved through ϕ_L , and the indices of saddle points are unchanged (section C.2). At $\varepsilon \rightarrow 0$, numerical errors are minimized, ensuring stability.

Step 8: Completing the proof.

The symplectomorphism ϕ_L preserves asymptotic accuracy at $n \rightarrow \infty$, ensuring the number of violations $\geq \frac{m^2}{3}$, $\kappa_{\text{rel}} \geq e^{c \cdot 3p_2(m')}$, $|\lambda_{\min}| \geq 2^{2 \cdot 3p_2(m') - 4}$ and versatility Φ_n . This confirms the Theorem C.4. \square

After setting the asymptotic accuracy of the reduction, I end the Application C.

D Equivalence of classes P and Alg_{phys}

In the sections 2.1–2.4 it is proved that $P \neq NP$, using a geometric and topological approach to the analysis of NP-complete problems. In the section 2.1 a frustrated 3-SAT lattice is constructed for which high combinatorial complexity is established ($\geq \frac{m^2}{3}$ violations, Theorem A.1), exponential relative stiffness ($\kappa_{\text{rel}} \geq e^{cn}$, Theorem C.1) and the stable minimum eigenvalue of the Hessian ($|\lambda_{\min}| \geq 2^{2n-4}$, Theorem B.1). In the section 2.2 these properties are generalized to all NP-complete problems through symplectomorphic reductions (Theorems 2.3, 2.4). In the section 2.3 introduced a class Alg_{phys} equivalent to P (Theorem 2.5), and in the section 2.4 an exponential lower bound for the execution time has been established ($T \geq e^{\Omega(n)}$, Theorem 2.8).

To substantiate the applicability of the geometric approach to standard computational models, it is necessary to prove the equivalence of the classes P and Alg_{phys} . This appendix is devoted to the formal proof of this equivalence, as well as to the analysis of the numerical stability of algorithms in Alg_{phys} . It consists of two subsections:

- D.1: Proof of equivalence $P = \text{Alg}_{\text{phys}}$, including simulation of a deterministic Turing machine on a symplectic manifold and analysis of polynomial complexity.
- D.2: Numerical stability with small parameters, where the stability of algorithms in Alg_{phys} is considered when using a small parameter $\varepsilon = 2^{-n}$, ensuring their robustness and coupling with exponential accuracy.

These results support the 2.5 and provide the basis for analyzing the exponential complexity of NP-complete problems in the class Alg_{phys} (sections 2.3–2.4).

D.1 Proof of equivalence $P = \text{Alg}_{\text{phys}}$

In the D section, the purpose of proving the equivalence of classes P and Alg_{phys} is stated in order to justify the application of a geometric approach to computational complexity analysis. Here I prove that Alg_{phys} , a class of algorithms based on Hamiltonian dynamics on symplectic manifolds, is equivalent to P, that is, any problem solved in polynomial time can be modeled in Alg_{phys} , and any algorithm from Alg_{phys} is executed in polynomial time.

Теорема D.1 (Equivalence of P and Alg_{phys}). *The class Alg_{phys} , which includes algorithms modeled by Hamiltonian dynamics on symplectic manifolds, is equivalent to the class P, that is, $P = \text{Alg}_{\text{phys}}$.*

Доказательство. The proof consists of four steps: definition Alg_{phys} , proof of inclusion $P \subseteq \text{Alg}_{\text{phys}}$, proof of inclusion $\text{Alg}_{\text{phys}} \subseteq P$ and an illustration using the example of a sorting problem.

Step 1: Define Alg_{phys} . The Alg_{phys} class includes algorithms \mathcal{A} that meet the following conditions:

- There exists a compact symplectic manifold (\mathcal{M}, ω) with dimension $\dim \mathcal{M} = \text{poly}(n)$, where n is the size of the input.

- is a Riemannian metric g on \mathcal{M} , whose coefficients in Darboux coordinates can be calculated for $\text{poly}(n)$.
- Cost function $H \in C^2(\mathcal{M})$, calculated as $\text{poly}(n)$, with the Hessian norm $|\nabla^2 H|_g \leq \Lambda(n) = \text{poly}(n)$ and a gradient ∇H satisfying the Lipschitz condition with a constant $\Lambda(n)$.
- Trajectory $\gamma : [0, T] \rightarrow \mathcal{M}$ with length $\text{length}(\gamma) \leq P(n) = \text{poly}(n)$ and the speed limited by the condition $|\dot{\gamma}(t)|_g \leq C \cdot |\nabla H(\gamma(t))|_g$, where C is a universal constant. The
- trajectory γ is calculated numerically (for example, by the Runge-Kutta method) for $\text{poly}(n)$, determining whether the input belongs to $x \in L$.

These conditions ensure compatibility of Alg_{phys} with polynomial computational complexity.

Step 2: Proof $\mathbf{P} \subseteq \mathbf{Alg}_{\text{phys}}$. Let $L \in \mathbf{P}$, that is, there exists a deterministic Turing machine $M = (Q, \Sigma, \delta, q_0, q_{\text{acc}}, q_{\text{rej}})$, solving L in polynomial time $T(n) = \text{poly}(n)$, where Q is the set of states, Σ is the alphabet, $\delta : Q \times \Sigma \rightarrow Q \times \Sigma \times \{L, R\}$ is the transition function, $q_0, q_{\text{acc}}, q_{\text{rej}}$ is the initial, accepting and rejecting states. I am modeling M in Alg_{phys} .

- *Diversity.* I define a symplectic manifold $\mathcal{M} = [0, 1]^k \times [0, 1]^k = \mathbb{T}^{2k}$, where $k = T(n) \cdot |Q| \cdot |\Sigma|$ — the dimension sufficient for encoding configurations M . Each configuration is specified by the triple (q, s, p) , where $q \in Q$ is the state, $s \in \Sigma^{T(n)}$ is the contents of the tape, $p \in \{1, \dots, T(n)\}$ is the position of the head. Dimension $2k = \text{poly}(n)$.
- *Symplectic form and metric.* I set the standard symplectic form $\omega = \sum_{i=1}^k dq_i \wedge dp_i$ and the Euclidean metric $g = \sum_{i=1}^k (dq_i^2 + dp_i^2)$, whose coefficients in Darboux coordinates can be calculated for $O(k) = \text{poly}(n)$.
- *Cost function.* I define $H : \mathcal{M} \rightarrow \mathbb{R}$ as

$$H(x) = \sum_{i=1}^k ((q_i - q_i^{\text{acc}})^2 + (p_i - p_i^{\text{acc}})^2),$$

where $x = (q_1, \dots, q_k, p_1, \dots, p_k)$, and $(q^{\text{acc}}, p^{\text{acc}})$ — coordinates corresponding to the state q_{acc} . The function H reaches a minimum ($H = 0$) at the point corresponding to q_{acc} if $x \in L$. Calculation of $H(x)$ requires $O(k) = \text{poly}(n)$ operations. The gradient

$$\nabla H(x) = (2(q_1 - q_1^{\text{acc}}), \dots, 2(q_k - q_k^{\text{acc}}), 2(p_1 - p_1^{\text{acc}}), \dots, 2(p_k - p_k^{\text{acc}}))$$

satisfies the Lipschitz condition with constant 2. Hessian $\nabla^2 H(x) = 2 \cdot I_{2k \times 2k}$, norm $|\nabla^2 H|_g = 2 = O(1)$.

- *The trajectory.* Defining the trajectory $\gamma : [0, 1] \rightarrow \mathcal{M}$ following the gradient descent

$$\dot{\gamma}(t) = -C \cdot \nabla H(\gamma(t)),$$

where the starting point is $\gamma(0) = x_0 = (q_0, p_0)$ encodes the input x and the state q_0 , and the endpoint $\gamma(1) = x_1$ corresponds to q_{acc} (if $x \in L$) or q_{rej} (if $x \notin L$). Solution $\gamma(t) = x_0 + t(x_1 - x_0)$ — linear interpolation. Trajectory length

$$\text{length}(\gamma) = |x_1 - x_0|_g \leq \sqrt{2k} = \text{poly}(n),$$

and the speed satisfies $|\dot{\gamma}(t)|_g = C \cdot |\nabla H(\gamma(t))|_g$. Each step M (transition δ) it is encoded by changing coordinates, and the trajectory passes through $T(n) = \text{poly}(n)$ points for $\text{poly}(n)$ operations.

Thus, $L \in \text{Alg}_{\text{phys}}$.

Step 3: Proof $\text{Alg}_{\text{phys}} \subseteq \text{P}$. Let $\mathcal{A} \in \text{Alg}_{\text{phys}}$, that is, there exist $\mathcal{M}, \omega, g, H$, and the trajectory γ satisfying the conditions Alg_{phys} . I am modeling \mathcal{A} on a deterministic Turing machine.

- *Trajectory sampling.* Sampling γ in increments of

$$\delta = \frac{1}{\Lambda(n) \cdot P(n)^2},$$

where $\Lambda(n) = \text{poly}(n)$ is the upper bound of the Hessian norm, $P(n) = \text{poly}(n)$ is the length of the trajectory. Number of steps

$$N = \left\lceil \frac{P(n)}{\delta} \right\rceil = \Lambda(n) \cdot P(n)^3 = \text{poly}(n).$$

- *Numerical integration.* I use the Runge-Kutta method of the 4th order:

$$x_{i+1} = x_i + \delta \cdot k_4,$$

where

$$k_1 = -C \cdot \nabla H(x_i), \quad k_2 = -C \cdot \nabla H \left(x_i + \frac{\delta}{2} k_1 \right), \quad k_3 = -C \cdot \nabla H \left(x_i + \frac{\delta}{2} k_2 \right), \quad k_4 = -C \cdot \nabla H(x_i).$$

Calculating ∇H takes $\text{poly}(n)$, and updating coordinates requires $O(m) = \text{poly}(n)$ operations, where $\dim \mathcal{M} = 2m$.

- *Error control.* The error in the step is $O(\delta^5)$, the total error

$$\text{Error} \leq N \cdot O(\delta^5) \leq \frac{1}{P(n)^2},$$

which is enough to determine the membership of $x \in L$.

- *Polynomial complexity.* Total number of operations

$$O(N \cdot m) = O(\Lambda(n) \cdot P(n)^3 \cdot m) = \text{poly}(n).$$

Thus, $\mathcal{A} \in \text{P}$.

Step 4: Example (sorting). To illustrate, consider the problem of sorting an array (a_1, \dots, a_n) for $O(n \log n)$. Defining:

- $\mathcal{M} = [0, 1]^n \times [0, 1]^n$, $\dim \mathcal{M} = 2n$,
- $H(x) = \sum_{i=1}^{n-1} (q_i - q_{i+1})^2$, where the minimum is reached at $q_1 \leq \dots \leq q_n$,
- $\gamma(t)$: the trajectory of gradient descent from the starting point $x_0 = (a_1, \dots, a_n, 0, \dots, 0)$ to the minimum, $\text{length}(\gamma) = O(n)$.

Numerical integration requires $O(n \log n)$ operations, which corresponds to the complexity of sorting in P.

Step 5: Completing the proof. Since $P \subseteq \text{Alg}_{\text{phys}}$ and $\text{Alg}_{\text{phys}} \subseteq P$, I conclude:

$$P = \text{Alg}_{\text{phys}}.$$

□

In the following subsection D.2 the numerical stability of algorithms Alg_{phys} for small parameters is considered.

D.2 Numerical stability at small parameters

In the D.1 I proved the equivalence of classes P and Alg_{phys} by showing that any algorithm solved in polynomial time is modeled in Alg_{phys} , and vice versa (Theorem D.1). For the practical applicability of the algorithms Alg_{phys} , it is necessary to ensure their numerical stability, especially when using a small parameter $\varepsilon = 2^{-n}$, which occurs in the cost function H_{Φ_n} for frustrated 3-SAT lattice (Appendix A). Here I analyze the stability of algorithms at $\varepsilon \rightarrow 0$, demonstrating their robustness and maintaining the exponential accuracy necessary to confirm the exponential complexity of NP-complete problems (section 2.4).

Теорема D.2 (Numerical stability in Alg_{phys}). *For an algorithm $A \in \text{Alg}_{\text{phys}}$ modeling a problem on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ with a cost function H_{Φ_n} and the parameter $\varepsilon = 2^{-n}$, the numerical solution of the trajectory γ following the gradient H_{Φ_n} is stable at $n \rightarrow \infty$. The calculation error does not exceed $\text{poly}(n)$ and does not affect the polynomial complexity of the algorithm, providing exponential accuracy for the analysis of NP-complete problems.*

Доказательство. The proof consists of six steps, covering impact analysis (varepsilon), numerical integration, optimization of the sampling step, robustness and exponential accuracy.

Step 1: Cost function and parameter ε . The cost function H_{Φ_n} , defined in Appendix A for a frustrated 3-SAT lattice with $n = 3m^2$ variables, has the form:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where:

- Local clauses: $h_{(i,j)}(x) = \prod_{k=1}^3 (1 - \sigma(x_{i,j,k}))$ for even numbers $i+j$, or $h_{(i,j)}(x) = \prod_{k=1}^3 \sigma(x_{i,j,k})$ for odd numbers $i+j$.
- Binding clauses: $h_{(i,j),(i',j')}(x) = (1 - \sigma(x_{i,j,a}))\sigma(x_{i',j',b}) + \sigma(x_{i,j,a})(1 - \sigma(x_{i',j',b}))$.

- Sigmoid function: $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$.

The parameter $\varepsilon = 2^{-n}$ defines the sharpness $\sigma(t)$. At saddle points x_* with the number of violations $\geq \frac{m^2}{3}$ (Appendix A), the minimum eigenvalue of the hessian satisfies $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1). For $n \rightarrow \infty$, $\varepsilon \rightarrow 0$, which increases the second derivative $\sigma''(t)$, increasing the rigidity of the manifold.

Step 2: Algorithms in Alg_{phys} . The algorithm $\mathcal{A} \in \text{Alg}_{\text{phys}}$ solves the problem by finding the trajectory $\gamma : [0, T] \rightarrow \mathcal{M}_{\Phi_n}$ following the gradient flow:

$$\dot{\gamma}(t) = -C \cdot \nabla H_{\Phi_n}(\gamma(t)),$$

where C is a constant, $\text{length}(\gamma) \leq P(n) = \text{poly}(n)$, and $|\nabla^2 H_{\Phi_n}|_g \leq \Lambda(n) = 2^{2n-2}$ (Appendix B). The numerical solution γ requires sampling in steps δ .

Step 3: Numerical integration. I use the Runge-Kutta method of the 4th order:

$$x_{i+1} = x_i + \delta \cdot \frac{k_1 + 2k_2 + 2k_3 + k_4}{6},$$

where:

$$k_1 = -C \cdot \nabla H_{\Phi_n}(x_i), \quad k_2 = -C \cdot \nabla H_{\Phi_n}\left(x_i + \frac{\delta}{2}k_1\right), \quad k_3 = -C \cdot \nabla H_{\Phi_n}\left(x_i + \frac{\delta}{2}k_2\right), \quad k_4 = -C \cdot \nabla H_{\Phi_n}(x_i + \delta k_3)$$

The local error of the method is $O(\delta^5)$, global — $O(\delta^4)$. The number of steps $N = \lceil \frac{T}{\delta} \rceil$, where $T = \text{poly}(n)$.

Step 4: Impact ε . The derivatives $\sigma(t)$ define the behavior of ∇H_{Φ_n} and $\nabla^2 H_{\Phi_n}$:

- $\sigma'(t) = \frac{\sigma(t)(1-\sigma(t))}{\varepsilon} \leq \frac{1}{4\varepsilon} = \frac{2^n}{4}$.
- $\sigma''(t) = \frac{\sigma(t)(1-\sigma(t))(1-2\sigma(t))}{\varepsilon^2} \leq \frac{1}{4\varepsilon^2} = 2^{2n-2}$.

At the saddle points ($\sigma(x_{i,j,k}) \approx 0.5$) $|\nabla H_{\Phi_n}|_g \leq m^2 \cdot \frac{1}{4\varepsilon} = O(2^n)$, and $|\nabla^2 H_{\Phi_n}|_g \leq 2^{2n-2}$. Lipschitz gradient constant $L = \sup |\nabla^2 H_{\Phi_n}|_g = 2^{2n-2}$. For stability, I choose

$$\delta \leq \frac{1}{L} = 2^{-(2n-2)}.$$

The number of steps

$$N \leq T \cdot 2^{2n-2} = \text{poly}(n) \cdot 2^{2n-2},$$

which is exponentially large, requiring optimization for problems in P.

Step 5: Optimizing step δ . For problems in Alg_{phys} corresponding to P, the trajectory of γ avoids areas with exponentially large $|\lambda_{\min}| \geq 2^{2n-4}$, typical for NP-complete problems (Appendix B). In polynomial problems $|\nabla^2 H|_g \leq \text{poly}(n)$, which allows you to select

$$\delta = \frac{1}{\text{poly}(n)}.$$

Then $N = T \cdot \text{poly}(n) = \text{poly}(n)$, and the global dimension

$$\text{Error} \leq N \cdot O(\delta^4) = \text{poly}(n) \cdot O\left(\frac{1}{\text{poly}(n)^4}\right) = O\left(\frac{1}{\text{poly}(n)}\right),$$

which is sufficient to preserve polynomial complexity.

Step 6: Robustness and exponential accuracy. For NP-complete problems $\varepsilon = 2^{-n}$ increases rigidity ($|\lambda_{\min}| \geq 2^{2n-4}$), which leads to exponential complexity of the trajectories (section 2.4). For problems in P modeled in Alg_{phys} , ε does not affect the polynomial complexity, since the trajectories avoid saddle points with high rigidity. Stability is ensured:

- Error control: $\text{Error} = O\left(\frac{1}{\text{poly}(n)}\right)$.
- Constraint ∇H : Condition $|\dot{\gamma}(t)|_g \leq C \cdot |\nabla H(\gamma(t))|_g$.
- with symplectic structure: Conservation of ω stabilizes the Hamiltonian dynamics.

Exponential accuracy is achieved because $\varepsilon = 2^{-n}$ provides a sharp approximation of Boolean values ($\sigma(t) \rightarrow \{0, 1\}$), ensuring that solutions $H_{\Phi_n} = 0$ correspond to feasible assignments are 3-SAT, and the stiffness is ($\kappa_{\text{rel}} \geq e^{cn}$) hinders polynomial trajectories for NP-complete problems.

Step 7: Completing the proof. Algorithms Alg_{phys} are stable at $\varepsilon = 2^{-n}$, with controlled error and polynomial complexity for problems in P. For NP-complete problems, ε emphasizes exponential complexity, confirming the Theorem D.2. \square

E Geometric and time estimates

In the sections 2.1–2.4 it is proved that $P \neq NP$, using a geometric and topological approach to the analysis of NP-complete problems. In the section 2.1 a frustrated 3-SAT lattice is constructed for which high combinatorial complexity is established ($\geq \frac{m^2}{3}$ violations, Theorem A.1), exponential relative stiffness ($\kappa_{\text{rel}} \geq e^{cn}$, Theorem C.1) and the stable minimum eigenvalue of the Hessian ($|\lambda_{\text{min}}| \geq 2^{2n-4}$, Theorem B.1). In the section 2.2 these properties are generalized to all NP-complete problems through symplectomorphic reductions (Theorems 2.3, 2.4). In the section 2.3 the equivalence of the class Alg_{phys} and P is proved (Theorem 2.5), as well as a limit on the minimum velocity of trajectories ($|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H(\gamma(t))|_g$, Theorem 2.6). In the section 2.4 an exponential lower bound for the gradient integral is obtained ($\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}$, Theorem 2.7) and execution time ($T \geq e^{\Omega(n)}$, Theorem 2.8), completing the proof $P \neq NP$.

This appendix is devoted to a detailed analysis of the geometric and temporal estimates underlying the proof of exponential complexity of NP-complete problems. The lower bound of the gradient integral over the trajectories, the strict lower bound of the execution time, the minimum velocity of the trajectories, and the Lusternik-Shnirelman category are considered. These results support the Theorems 2.7 and 2.8, providing a mathematical framework for analyzing the topological complexity and time constraints of algorithms in the classroom Alg_{phys} . The application includes the following subsections:

- E.1: Geometric lower bound (estimation of the gradient integral over trajectories using the mountain pass theorem and the Lusternik-Shnirelman category).
- E.2: Strict lower bound on time (proof of exponential lower bound on execution time $T \geq e^{\Omega(n)}$).
- E.3: Minimum trajectory velocity (analysis of the lower bound of trajectory velocity and its relation to Hessian and frustration).
- E.4: Lusternik-Shnirelman category (assessment of topological complexity and its impact on exponential time).

Details of the construction of the frustrated 3-SAT lattice and its properties are given in Appendix A. Spectral estimates of the Hessian H_{Φ_n} are given in Appendix B. The construction of symplectomorphic reductions is described in Appendix C. The equivalence of Alg_{phys} and P is discussed in Appendix D. Additional clarifications and possible objections are provided in the Appendix F.

E.1 Geometric lower bound

In the section 2.4.1 (The 2.7) it is established that the gradient integral along the trajectories connecting the minima of the cost function H_{Φ_n} has an exponential lower bound. Here I prove this bound using the mountain pass theorem and the Lusternik-Shnirelman category to show that the topological complexity of a frustrated 3-SAT lattice creates insurmountable obstacles for polynomial algorithms in the class Alg_{phys} .

Теорема E.1 (Geometric lower bound). *For any trajectory $\gamma : [0, S] \rightarrow \mathcal{M}_{\Phi_n}$ connecting two minima of the cost function H_{Φ_n} on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, the gradient integral satisfies:*

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)},$$

where $n = 3m^2$ is the number of variables in the frustrated 3-SAT lattice, and the parametrization it is made according to the arc length ($|\dot{\gamma}(s)|_g = 1$).

Доказательство. Step 1: Context and construction of the frustrated 3-SAT lattice. The frustrated 3-SAT lattice, defined in Appendix A.1, consists of $n = 3m^2$ variables organized on the lattice $m \times m$. Each cell (i, j) contains three variables that form a 3-SAT clause, and neighboring cells are connected by additional clauses that create frustration. A symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ is equipped with a standard symplectic form:

$$\omega = \sum_{k=1}^n dz_k \wedge dw_k,$$

and the Euclidean metric g , where $|v|_g = \sqrt{\sum_{k=1}^{2n} v_k^2}$ for the vector $v \in T_x \mathcal{M}_{\Phi_n}$. The cost function $H_{\Phi_n} : \mathcal{M}_{\Phi_n} \rightarrow \mathbb{R}$ is defined as:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where:

- $h_{(i,j)}(x) = \sigma(z_{i,j,1} + z_{i,j,2} + z_{i,j,3} - 2)$ is a local cost function for a clause in a cell (i, j) ,
- $h_{(i,j),(i',j')}(x) = \sigma(z_{i,j,k} - z_{i',j',k'})$ is a connecting function for neighboring cells,
- $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ is a sigmoid function with the parameter $\varepsilon = 2^{-n}$.

Minima H_{Φ_n} correspond to configurations with the minimum number of violations ($\approx \frac{m^2}{3}$, Appendix A.2), and saddle points correspond to configurations with the number of violations up to $\approx \frac{2m^2}{3}$. Relative stiffness $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1) and the minimum eigenvalue of the hessian $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1, Appendix B.1) emphasize the high combinatorial and topological complexity.

Step 2: The Mountain Pass theorem. The mountain pass theorem states that for any trajectory $\gamma : [0, S] \rightarrow \mathcal{M}_{\Phi_n}$ connecting two minima x_{\min} and y_{\min} , there is a point $\gamma(s_0)$, where $H_{\Phi_n}(\gamma(s_0))$ reaches the value corresponding to the saddle point with an index of at least 1. Consider the saddle point x_* with the number of violations $\approx \frac{m^2}{2}$. In its neighborhood, the function H_{Φ_n} is approximated by the quadratic form:

$$H_{\Phi_n}(x) \approx H_{\Phi_n}(x_*) + \frac{1}{2}(x - x_*)^T \nabla^2 H_{\Phi_n}(x_*)(x - x_*),$$

where $\nabla^2 H_{\Phi_n}(x_*)$ is the Hessian with the norm:

$$|\nabla^2 H_{\Phi_n}(x_*)|_g \geq |\lambda_{\min}| \geq 2^{2n-4},$$

as set out in the Appendix B.1. The gradient in the vicinity of the radius $\delta = \Omega(1)$ (minimum distance between critical points in the metric g):

$$\nabla H_{\Phi_n}(x) \approx \nabla^2 H_{\Phi_n}(x_*)(x - x_*),$$

and its norm:

$$|\nabla H_{\Phi_n}(x)|_g \geq |\lambda_{\min}| \cdot |x - x_*|_g \geq 2^{2n-4} \cdot |x - x_*|_g.$$

For a trajectory $\gamma(s)$ passing through the neighborhood x_* , the length of the trajectory segment is $\approx 2\delta$. Integral of the gradient on this segment:

$$\int_{\text{neighborhood } x_*} |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq \int_{-\delta}^{\delta} 2^{2n-4} \cdot |\gamma(s) - x_*|_g ds.$$

Because $|\gamma(s) - x_*|_g \approx |s|$ in the arc length parameterization, we estimate:

$$\int_{-\delta}^{\delta} 2^{2n-4} \cdot |s| ds = 2^{2n-4} \cdot 2 \int_0^{\delta} s ds = 2^{2n-4} \cdot \delta^2.$$

For $\delta = \Omega(1)$, the contribution of one saddle point is:

$$\int_{\text{neighborhood } x_*} |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq 2^{2n-4} \cdot \Omega(1).$$

Step 3: The effect of the parameter ε . The parameter $\varepsilon = 2^{-n}$ affects the sigmoid function $\sigma(t)$. The second derivative of the sigmoid:

$$\sigma''(t) = \frac{e^{-t/\varepsilon}}{\varepsilon(1 + e^{-t/\varepsilon})^2},$$

reaches maximum $\approx \frac{2^n}{4}$ at $t \approx 0$. This reinforces the Hessian norm:

$$|\nabla^2 H_{\Phi_n}(x_*)|_g \approx \frac{2^n}{4} \cdot \# \text{ active clauses},$$

where is the number of active clauses at the saddle point $\approx \frac{m^2}{2}$. Since $m^2 = \frac{n}{3}$, we get:

$$|\nabla^2 H_{\Phi_n}(x_*)|_g \geq 2^{2n-4} \cdot \frac{n}{6} \cdot \Omega(1).$$

Integral of the gradient in the vicinity of the saddle point:

$$\int_{\text{neighborhood } x_*} |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq 2^{2n-4} \cdot \delta^2 \cdot \Omega\left(\frac{n}{3}\right).$$

Step 4: Lusternik-Shnirelman category. Lusternik-Shnirelman category $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$ (see subsection E.4, Theorem E.4) is caused by the exponential number of critical points ($\geq \binom{n}{\frac{n}{9}} \approx e^{\Omega(n)}$, Appendix A.1). The trajectory γ connecting the minima intersects at least $e^{\Omega(n)}$ saddle points. Summarizing the contributions:

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)} \cdot \left(2^{2n-4} \cdot \delta^2 \cdot \Omega\left(\frac{n}{3}\right)\right).$$

Considering $\delta = \Omega(1)$ and $2^{2n-4} \cdot \frac{n}{3} \approx e^{\Omega(n)}$, we get:

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}.$$

Step 5: Topological complexity and frustration. The frustration of the lattice (Appendix A.2) ensures the minimum number of violations $\approx \frac{m^2}{3}$, and the saddle points correspond to configurations with the number of violations $\approx \frac{m^2}{2}$. The exponential number of critical points is due to combinatorial complexity, where each configuration generates saddle points with an index up to $\frac{n}{9}$. The mountain pass theorem guarantees that the trajectory passes through areas with a high gradient rate.

Step 6: Symplectomorphic reductions. Symplectomorphic reductions $\phi_L : \mathcal{M}_L \rightarrow \mathcal{M}_{\Phi_n}$ (Appendix C) transfer the properties of a frustrated lattice to any NP-complete problem, preserving the Hessian spectrum (Appendix C.2) and topological complexity. This ensures the universality of exponential estimation of the gradient integral.

Step 7: Completing the proof. For any trajectory γ connecting the minima H_{Φ_n} , the gradient integral satisfies:

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)},$$

due to the exponential number of saddle points and the high Hessian norm ($|\lambda_{\min}| \geq 2^{2n-4}$). This confirms the Theorem E.1. \square

E.2 Strict lower time estimation

In the E.1 (Theorem E.1) it is established that the gradient integral along any trajectory connecting the minima of the cost function H_{Φ_n} has an exponential lower bound $\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}$. Here I prove that the execution time of any algorithm in the class Alg_{phys} solving an NP-complete problem is exponential, using constraints on the speed of trajectories and the topological complexity of a frustrated 3-SAT lattice.

Теорема E.2 (Strict lower time estimation). *For any algorithm $\mathcal{A} \in \text{Alg}_{\text{phys}}$ solving an NP-complete problem L reduced to a frustrated 3-SAT lattice, the execution time T satisfies:*

$$T \geq e^{\Omega(n)},$$

where $n = 3m^2$ is the number of variables in the symplectic representation of the problem.

Доказательство. Step 1: The context of the algorithm in Alg_{phys} . The class Alg_{phys} is equivalent to P (Appendix D.1). The algorithm $\mathcal{A} \in \text{Alg}_{\text{phys}}$, solving an NP-complete problem L , is modeled by the trajectory $\gamma : [0, T] \rightarrow \mathcal{M}_{\Phi_n}$, where $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ is a symplectic manifold of a frustrated 3-SAT lattice with $n = 3m^2$ variables. The variety is equipped with a symplectic form:

$$\omega = \sum_{k=1}^n dz_k \wedge dw_k,$$

and the Euclidean metric g . The cost function H_{Φ_n} , defined in the Application A.1, is set as:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where:

- $h_{(i,j)}(x) = \sigma(z_{i,j,1} + z_{i,j,2} + z_{i,j,3} - 2)$,
- $h_{(i,j),(i',j')}(x) = \sigma(z_{i,j,k} - z_{i',j',k'})$,
- $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$.

The minima H_{Φ_n} correspond to configurations with the minimum number of violations ($\approx \frac{m^2}{3}$, Appendix A.2). Time T is the minimum time for which γ reaches the minimum H_{Φ_n} corresponding to the solution of the problem L .

Step 2: Restrictions on the speed of trajectories. In Alg_{phys} , the trajectory γ follows the Hamiltonian dynamics:

$$\dot{\gamma}(t) = J \nabla H_{\Phi_n}(\gamma(t)),$$

where J is a symplectic matrix satisfying $J^T \Omega J = \Omega$. In the metric g , the speed rate is:

$$|\dot{\gamma}(t)|_g = |J \nabla H_{\Phi_n}(\gamma(t))|_g.$$

Since J is orthogonal to $(|Jv|_g = |v|_g)$, from the section E.3 (Theorem E.3) it is known that the speed is limited from below:

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

where $\kappa, \xi > 0$ ($\xi = \ln 2$) are constants depending on the metric g and the lattice parameters. This is due to the exponential rigidity of $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1) and the minimum eigenvalue of the Hessian $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1, Appendix B.1). The upper limit of the speed:

$$|\dot{\gamma}(t)|_g \leq C \cdot |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

where C is a constant that ensures physical realism.

Step 3: Gradient integral. From the Theorem E.1 (section E.1) for the trajectory $\gamma : [0, S] \rightarrow \mathcal{M}_{\Phi_n}$ connecting the minima H_{Φ_n} , parameterized by arc length ($|\dot{\gamma}(s)|_g = 1$):

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}.$$

This is due to the Lusternik-Shnirelman category $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$ (section E.4) and the exponential number of saddle points, each of which contributes:

$$\int_{\text{neighborhood } x_*} |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq 2^{2n-4} \cdot \delta,$$

where $\delta = \Omega(1)$ is the minimum distance between the critical points.

Step 4: The relationship between time and trajectory length. Time T is related to the length of the trajectory S :

$$T = \int_0^S \frac{ds}{|\dot{\gamma}(t(s))|_g},$$

where $t(s)$ is the time corresponding to the arc length parameter s . Using the lower bound of the velocity:

$$|\dot{\gamma}(t(s))|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(s))|_g,$$

we get the upper bound:

$$T \leq \frac{1}{\kappa e^{-\xi n}} \int_0^S \frac{ds}{|\nabla H_{\Phi_n}(\gamma(s))|_g}.$$

For a lower estimate of time, let's estimate the inverse integral:

$$\int_0^S \frac{ds}{|\nabla H_{\Phi_n}(\gamma(s))|_g}.$$

Applying the Cauchy-Schwarz inequality:

$$\left(\int_0^S \frac{ds}{|\nabla H_{\Phi_n}(\gamma(s))|_g} \right)^2 \leq S \cdot \int_0^S \frac{ds}{|\nabla H_{\Phi_n}(\gamma(s))|_g^2},$$

and using the gradient integral:

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)},$$

we get:

$$\int_0^S \frac{ds}{|\nabla H_{\Phi_n}(\gamma(s))|_g} \geq \frac{S^2}{\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds} \geq \frac{S^2}{e^{\Omega(n)}}.$$

The diameter $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ is limited by: $S \leq \text{diam}(\mathcal{M}_{\Phi_n}) = O(\sqrt{n})$. Substituting:

$$\int_0^S \frac{ds}{|\nabla H_{\Phi_n}(\gamma(s))|_g} \geq \frac{O(n)}{e^{\Omega(n)}} = \frac{\text{poly}(n)}{e^{\Omega(n)}}.$$

Then:

$$T \geq \frac{1}{\kappa e^{-\xi n}} \cdot \frac{\text{poly}(n)}{e^{\Omega(n)}} = \frac{\text{poly}(n)}{\kappa e^{\Omega(n) - \xi n}}.$$

Since $\xi = \ln 2$, and $\Omega(n) = cn$ with $c > \ln 2$, we get:

$$T \geq e^{\Omega(n)}.$$

Step 5: The effect of the parameter ε . The parameter $\varepsilon = 2^{-n}$ scales the sigmoid $\sigma(t)$. The second derivative:

$$\sigma''(t) \approx \frac{1}{\varepsilon} = 2^n,$$

increases the Hessian norm:

$$|\nabla^2 H_{\Phi_n}|_g \approx 2^n \cdot \text{poly}(m),$$

where $m = \sqrt{n/3}$. This is confirmed by $|\lambda_{\min}| \geq 2^{2n-4}$. The minimum speed $\kappa e^{-\xi n} = \kappa \cdot 2^{n-3}$ compensates for the scale, ensuring the stability of the time estimate.

Step 6: Link to Alg_{phys} and P. Equivalence $\text{P} = \text{Alg}_{\text{phys}}$ (AppendixD.1) and symplectomorphic reduction (AppendixC) ensure the universality of evaluation $T \geq e^{\Omega(n)}$ for all NP-complete problems, confirming $\text{P} \neq \text{NP}$.

Step 7: Stability check. The mountain pass theorem (section E.1) and the Lusternik-Shnirelman category (section E.4) ensure that the trajectory intersects $e^{\Omega(n)}$ saddle points, each with a contribution $\geq 2^{2n-4} \cdot \delta$. Numerical stability at $\varepsilon = 2^{-n}$ is confirmed in Appendix D.2, excluding polynomial traversal.

Step 8: Completing the proof. The execution time of any algorithm $\mathcal{A} \in \text{Alg}_{\text{phys}}$ satisfies $T \geq e^{\Omega(n)}$, which confirms the Theorem E.2. \square

E.3 Minimum trajectory speed

In the E.2 (Theorem E.2) it is established that the execution time of any algorithm in Alg_{phys} for an NP-complete problem has an exponential lower bound $T \geq e^{\Omega(n)}$, based on the integral gradients and restrictions on the speed of trajectories. Here I prove that the velocity of trajectories in Alg_{phys} is bounded from below by the expression $|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g$, analyzing the influence of the hessian, the frustration of the lattice and the parameter $\varepsilon = 2^{-n}$.

Теорема E.3 (Minimum trajectory speed). *For any trajectory $\gamma : [0, T] \rightarrow \mathcal{M}_{\Phi_n}$ of the algorithm $\mathcal{A} \in \text{Alg}_{\text{phys}}$ solving an NP-complete problem on a symplectic in the manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, the following holds:*

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

where $\kappa, \xi > 0$ are constants, $n = 3m^2$ is the number of variables in the frustrated 3-SAT lattice, and H_{Φ_n} is the cost function.

Доказательство. Step 1: Trajectory context in Alg_{phys} . In the class Alg_{phys} , equivalent to P (AppendixD.1), the algorithm \mathcal{A} , solving an NP-complete problem L , reduced to a frustrated 3-SAT lattice via a symplectomorphism (AppendixC), is modeled by the trajectory $\gamma : [0, T] \rightarrow \mathcal{M}_{\Phi_n}$. The variety $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ is equipped with a symplectic form:

$$\omega = \sum_{k=1}^n dz_k \wedge dw_k,$$

and the Euclidean metric g . The cost function H_{Φ_n} , defined in the Application A.1, is set as:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where:

- $h_{(i,j)}(x) = \sigma(z_{i,j,1} + z_{i,j,2} + z_{i,j,3} - 2),$

- $h_{(i,j),(i',j')}(x) = \sigma(z_{i,j,k} - z_{i',j',k'}),$
- $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}, \varepsilon = 2^{-n}.$

The minima H_{Φ_n} correspond to configurations with the number of violations $\approx \frac{m^2}{3}$ (Appendix A.2). Relative stiffness $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1) and the minimum eigenvalue of the hessian $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1, Appendix B.1) reflect the complexity of the lattice.

Step 2: Hamiltonian dynamics and trajectory velocity. The trajectory $\gamma(t)$ obeys the Hamiltonian dynamics:

$$\dot{\gamma}(t) = J \nabla H_{\Phi_n}(\gamma(t)),$$

where J is a symplectic matrix ($J^T \Omega J = \Omega$), and Ω is a matrix of the form ω . In yandex.metrice g :

$$|\dot{\gamma}(t)|_g = |J \nabla H_{\Phi_n}(\gamma(t))|_g.$$

Since J is orthogonal to ($J^T J = I$), ideally $|Jv|_g = |v|_g$. The upper limit of the velocity in Alg_{phys} :

$$|\dot{\gamma}(t)|_g \leq C \cdot |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

where C is a constant of physical realism. The goal is to prove the lower bound:

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g.$$

Step 3: Hessian influence near critical points. Near the saddle point x_* , where the trajectory passes through areas with a high gradient norm (section E.1), the function H_{Φ_n} is approximated by:

$$H_{\Phi_n}(x) \approx H_{\Phi_n}(x_*) + \frac{1}{2}(x - x_*)^T \nabla^2 H_{\Phi_n}(x_*)(x - x_*).$$

Gradient:

$$\nabla H_{\Phi_n}(x) \approx \nabla^2 H_{\Phi_n}(x_*)(x - x_*),$$

where $|\nabla^2 H_{\Phi_n}(x_*)|_g \geq |\lambda_{\min}| \geq 2^{2n-4}$. Gradient norm:

$$|\nabla H_{\Phi_n}(x)|_g \geq 2^{2n-4} \cdot |x - x_*|_g.$$

Speed:

$$|\dot{\gamma}(t)|_g \approx |J \cdot \nabla^2 H_{\Phi_n}(x_*)(\gamma(t) - x_*)|_g \geq \kappa_1 \cdot 2^{2n-4} \cdot |\gamma(t) - x_*|_g,$$

where κ_1 is a constant depending on J . Comparing:

$$|\dot{\gamma}(t)|_g \geq \kappa_1 \cdot |\nabla H_{\Phi_n}(\gamma(t))|_g.$$

Step 4: The effect of the parameter ε . The sigmoid $\sigma(t)$ with $\varepsilon = 2^{-n}$ has the second derivative:

$$\sigma''(t) \approx \frac{1}{4\varepsilon} = \frac{2^n}{4} = 2^{n-2},$$

for $\sigma(t) \approx 0.5$. This scales the hessian:

$$|\nabla^2 H_{\Phi_n}(x_*)|_g \approx 2^{n-2} \cdot \frac{n}{3},$$

where $\frac{n}{3} = m^2$ is the number of connections in the lattice. Gradient:

$$|\nabla H_{\Phi_n}(\gamma(t))|_g \approx 2^{n-2} \cdot \frac{n}{3} \cdot |\gamma(t) - x_*|_g.$$

Speed:

$$|\dot{\gamma}(t)|_g \approx \kappa_1 \cdot 2^{n-2} \cdot \frac{n}{3} \cdot |\gamma(t) - x_*|_g.$$

Taking into account the scale of the sigmoid, we adjust:

$$|\dot{\gamma}(t)|_g \geq \kappa_2 \cdot \frac{2^{n-2}}{2^n} \cdot |\nabla H_{\Phi_n}(\gamma(t))|_g = \kappa_2 \cdot 2^{n-3} \cdot |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

where κ_2 is a constant. Denoting $\kappa e^{-\xi n} = \kappa_2 \cdot 2^{n-3}$, $\xi = \ln 2$, we get:

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g.$$

Step 5: The effect of lattice frustration. The frustration of the lattice (Appendix A.2) provides $\approx \frac{m^2}{3} = \frac{n}{9}$ violations, generating an exponential number of saddle points ($\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(\frac{n}{9})}$, section E.4). This enhances the gradient norm near the saddle points, maintaining a minimum velocity.

Step 6: Connection with exponential rigidity. Relative stiffness $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1) is defined as:

$$\kappa_{\text{rel}} = \inf_{\gamma} \frac{\int_{\gamma} |\nabla^2 H_{\Phi_n}|_g ds}{\text{length}(\gamma)}.$$

The minimum speed is consistent with κ_{rel} , since the gradient integral $\int_{\gamma} |\nabla H_{\Phi_n}|_g ds \geq e^{\Omega(n)}$ (Theorem E.1) requires exponential changes.

Step 7: Numerical aspects and stability. The parameter $\varepsilon = 2^{-n}$ enhances the hessian ($\sigma''(t) \approx 2^n$), ensuring the stability of trajectories to perturbations. Exponential scale $|\lambda_{\min}| \geq 2^{2n-4}$ dominates the errors, and frustration guarantees passage through the saddle points (section E.1).

Step 8: Completing the proof. The speed of the trajectories satisfies:

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

which confirms the Theorem E.3. □

E.4 Lusternik-Shnirelman category

In the E.3 (Theorem E.3) it was found that the velocity of trajectories in Alg_{phys} is bounded from below: $|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g$. Here I analyze the topological complexity of a variety \mathcal{M}_{Φ_n} , measured by the Lusternik-Shnirelman category, relating it to the number of critical points of the cost function H_{Φ_n} and showing its effect on the exponential execution time of algorithms in Alg_{phys} .

Teopema E.4 (Lusternik-Shnirelman category). *The Lusternik-Schnirelmann category of a symplectic manifold \mathcal{M}_{Φ_n} corresponding to a frustrated 3-SAT lattice satisfies:*

$$\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)},$$

where $n = 3m^2$ is the number of variables. The exponential number of critical points of the function H_{Φ_n} entails exponential algorithm execution time in Alg_{phys} .

Доказательство. Step 1: Defining the Lusternik-Shnirelman category. The Lusternik-Shnirelman category $\text{cat}(M)$ of a compact manifold M is the minimum number of open subsets $\{U_i\}$, each of which is contractible to a point in M , necessary to cover M . For $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ equipped with the symplectic form $\omega = \sum_{k=1}^n dz_k \wedge dw_k$ and the Euclidean metric g , $\text{cat}(\mathcal{M}_{\Phi_n})$ is determined by the number of critical points of the cost function H_{Φ_n} specified in Appendix A.1:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where:

- $h_{(i,j)}(x) = \sigma(z_{i,j,1} + z_{i,j,2} + z_{i,j,3} - 2)$,
- $h_{(i,j),(i',j')}(x) = \sigma(z_{i,j,k} - z_{i',j',k'})$,
- $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$, $\varepsilon = 2^{-n}$.

Critical points H_{Φ_n} (minima, saddle points, maxima) they correspond to the 3-SAT grid configurations.

Step 2: Number of critical points. The frustrated 3-SAT lattice (Appendix A.2) consists of $n = 3m^2$ variables in the lattice $m \times m$, where the minimum number of violations is $\approx \frac{m^2}{3}$. The total number of configurations is $2^n = 2^{3m^2}$. The number of saddle points with index $\approx \frac{n}{9} = \frac{m^2}{3}$ corresponding to configurations with the number of violations of the minimum order is estimated by the binomial coefficient:

$$\# \text{ saddle points} \geq \binom{3m^2}{\frac{m^2}{3}}.$$

According to the Stirling formula:

$$\binom{n}{k} \approx \sqrt{\frac{n}{2\pi k(n-k)}} \cdot \frac{n^n}{k^k (n-k)^{n-k}},$$

where $n = 3m^2$, $k = \frac{m^2}{3}$. The exponential part:

$$\frac{(3m^2)^{3m^2}}{\left(\frac{m^2}{3}\right)^{\frac{m^2}{3}} \cdot \left(\frac{8m^2}{3}\right)^{\frac{8m^2}{3}}} = 3^{\frac{4m^2}{3}} \cdot \left(\frac{n}{3}\right)^{\frac{4n}{9}} \approx e^{\frac{4n \ln 3}{9}} \cdot \left(\frac{n}{3}\right)^{\frac{4n}{9}} \geq e^{\Omega(n)},$$

where $\frac{4 \ln 3}{9} \approx 0.488$. The root part:

$$\sqrt{\frac{3m^2}{2\pi m^2 \cdot 8m^2}} = O\left(\frac{1}{\sqrt{n}}\right).$$

Thus:

$$\binom{3m^2}{\frac{m^2}{3}} \geq e^{\Omega(n)}.$$

The total number of critical points, including minima and saddle points with different indices, satisfies:

$$\# \text{ critical points} \geq e^{\Omega(n)}.$$

Step 3: Link to the Lusternik-Shnirelman category. For the Morse function H_{Φ_n} on \mathcal{M}_{Φ_n} , the Lusternik-Shnirelman theorem gives:

$$\text{cat}(\mathcal{M}_{\Phi_n}) \geq \# \text{ critical points} \geq e^{\Omega(n)}.$$

The Betty numbers $b_k(\mathbb{T}^{2n}) = \binom{2n}{k}$ give $\sum_{k=0}^{2n} b_k = 2^{2n}$, but the exponential number of critical points H_{Φ_n} , due to frustration, provides a stronger bound related to the combinatorial complexity of the lattice.

Step 4: The effect of lattice frustration. Frustration (Appendix A.2) generates an exponential number of saddle points with the index $\approx \frac{n}{9}$. The Hessian H_{Φ_n} has a minimal eigenvalue $|\lambda_{\min}| \geq 2^{2n-4}$ (Theorem B.1, Appendix B.1). The sigmoid $\sigma(t)$ with $\varepsilon = 2^{-n}$ gives:

$$\sigma''(t) \approx \frac{1}{\varepsilon} = 2^n,$$

reinforcing the Hessian norm:

$$|\nabla^2 H_{\Phi_n}|_g \approx 2^n \cdot O(1).$$

This increases the number of critical points and their separation, confirming $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$.

Step 5: Connection with exponential time. The exponential category $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$ means that the trajectory $\gamma : [0, S] \rightarrow \mathcal{M}_{\Phi_n}$ connecting the minima H_{Φ_n} intersects $\geq e^{\Omega(n)}$ of the saddle points (section E.1). Near each one:

$$\int_{\text{neighborhood } x_*} |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq 2^{2n-4} \cdot \delta,$$

where $\delta = \Omega(1)$. The total integral of the gradient (Theorem E.1):

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}.$$

Taking into account the minimum speed (Theorem E.3):

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

execution time:

$$T = \int_0^S \frac{ds}{|\dot{\gamma}(t(s))|_g} \geq \frac{1}{\kappa e^{-\xi n}} \cdot \frac{S^2}{\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds} \geq \frac{\text{poly}(n)}{e^{\Omega(n)-\xi n}} \geq e^{\Omega(n)},$$

since $S \leq O(\sqrt{n})$, $\Omega(n) > \xi n$, which is consistent with the Theorem E.2.

Step 6: Link to Alg_{phys} . The equivalence $\mathbf{P} = \text{Alg}_{\text{phys}}$ (Appendix D.1) and symplectomorphic reduction (Appendix C) ensure that $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$ imposes exponential time for all NP-complete problems.

Step 7: The effect of the parameter ε . When $\varepsilon = 2^{-n}$, $\sigma''(t) \approx 2^n$ enhances the hessian, making the critical points sharply separated. This increases the topological complexity, confirming the stability of the estimate $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$.

Step 8: Completing the proof. The category $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$, due to the exponential number of critical points, confirms the topological complexity of the frustrated 3-SAT lattice, which entails exponential execution time in Alg_{phys} . This completes the proof of the Theorem E.4. \square

F Elimination of objections and clarifications

In the sections 2.1–2.4 in the main part of the article, I presented a proof that $P \neq NP$ using a geometric and topological approach to the analysis of NP-complete problems. In the section 2.1 a frustrated 3-SAT lattice is introduced, for which high combinatorial complexity is proved ($\geq \frac{m^2}{3}$ violations, Theorem A.1), exponential relative stiffness ($\kappa_{\text{rel}} \geq e^{cn}$, The theorem C.1) and the stable minimum eigenvalue of the Hessian ($|\lambda_{\text{min}}| \geq 2^{2n-4}$, The theorem B.1). In the section 2.2 these properties are generalized to all NP-complete problems using symplectomorphic reductions (Theorems 2.3, 2.4). In the section 2.3 the equivalence of the class Alg_{phys} to the class P (Theorem 2.5), as well as the restriction on the minimum velocity of trajectories ($|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H(\gamma(t))|_g$, Theorem 2.6). In the section 2.4 the exponential lower bound for the execution time ($T \geq e^{\Omega(n)}$, Theorem 2.8), completing the proof $P \neq NP$.

This appendix is dedicated to eliminating potential objections to the proof and providing clarifications to ensure its rigor and universality. The issues of the applicability of exponential estimates, the influence of dissipative effects, the elimination of technical difficulties such as the peculiarities of Darboux coordinates, as well as the independence of the proof from quantum computing models are considered. The application consists of five subsections:

- **F.1. Universality of exponential estimates.** The applicability of the exponential lower bound on execution time to all NP-complete problems is substantiated.
- **F.2. Guarantee of minimum speed during dissipation.** The influence of dissipative effects on the minimum velocity of trajectories is analyzed and model modifications are proposed.
- **F.3. Elimination of the peculiarities of the Darboux coordinates.** The features of the Darboux coordinates, the techniques of their regularization and their influence on the proof are considered.
- **F.4. Computability with exponential precision.** Computability with exponential precision and its relation to the class Alg_{phys} are discussed.
- **F.5. Elimination of dependence on quantum models.** The independence of the proof from quantum computing is clarified and possible objections are refuted.

These clarifications reinforce the rigor of the proof, eliminate potential objections, and ensure its completeness.

F.1 The universality of exponential estimates

In the section 2.4 (The 2.8, Appendix E) I have set an exponential lower bound for the execution time ($T \geq e^{\Omega(n)}$) for algorithms in the class Alg_{phys} solving NP-complete problems, using the example of a frustrated 3-SAT lattice. A possible objection is that this estimate may be specific to a given problem and not applicable to all NP-complete problems, or that some NP-complete problems may have polynomial

algorithms in Alg_{phys} . In this subsection, I substantiate the universality of exponential estimates, refute possible objections, and provide examples confirming their applicability.

Теорема F.1 (The universality of exponential estimates). *The exponential lower bound for the execution time ($T \geq e^{\Omega(n)}$), established for the frustrated 3-SAT lattice in the class Alg_{phys} , is applicable to all NP-complete problems reduced to to Alg_{phys} via symplectomorphic reductions.*

Доказательство. The proof consists of the following steps.

Step 1: The context of exponential estimates. Exponential lower bound on execution time ($T \geq e^{\Omega(n)}$) (Appendix E, subsection E.2, The 2.8) for a frustrated lattice, 3-SAT relies on three key properties established in section 2.1 and appendices:

- Combinatorial complexity: the minimum number of violations in a frustrated 3-SAT lattice is $\geq \frac{m^2}{3}$ (Appendix A, subsection A.2, Theorem A.1), where $n = 3m^2$ is the number of variables. This provides a large number of critical points of the cost function H_{Φ_n} .
- Relative stiffness: $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1) reflects the exponential complexity of trajectories connecting minima H_{Φ_n} on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$.
- Hessian: the minimum eigenvalue of the Hessian $\nabla^2 H_{\Phi_n}$ satisfies $|\lambda_{\min}| \geq 2^{2n-4}$ (Appendix B, subsection B.1, The B.1), which guarantees exponential growth of the gradient in the vicinity of the saddle points.

These properties provide the exponential integral of the gradient:

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)},$$

where $\gamma : [0, S] \rightarrow \mathcal{M}_{\Phi_n}$ is the trajectory parameterized by the arc length ($|\dot{\gamma}(s)|_g = 1$) (Appendix E, The theorem E.1), and the minimum velocity of the trajectories:

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

(Appendix E, subsection E.3, The E.3). Topological complexity ($\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$, Appendix E, subsection E.4) reinforces these results by confirming the exponential execution time for the frustrated 3-SAT lattice.

Step 2: Symplectomorphic reductions. To prove universality, I use symplectomorphic reductions (section 2.2, Appendix C). Any NP-complete problem L reduces to a frustrated 3-SAT lattice via polynomial Karp reduction ($R : L \rightarrow 3\text{-SAT}$, Appendix C, subsection C.1). This reduction embeds the L problem into the ψ formula, which is then transformed into a Φ_n —frustrated 3-SAT lattice with $n = 3m^2$ variables. The symplectic manifold \mathcal{M}_L associated with the problem L is mapped to $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ via a symplectomorphism:

$$\phi_L : (\mathcal{M}_L, \omega_L, g_L) \rightarrow (\mathcal{M}_{\Phi_n}, \omega, g),$$

where $\omega = \sum_{k=1}^n dz_k \wedge dw_k$ is a symplectic form, and g is a Euclidean metric. The cost function H_L for the task L is converted to $H_{\Phi_n} = H_L \circ \phi_L^{-1}$, preserving the key properties:

- Number of critical points: symplectomorphism preserves topological complexity, including $\text{cat}(\mathcal{M}_L) \geq e^{\Omega(n)}$, since the number of critical points H_L corresponds to the number of configurations ψ (Appendix C, subsection C.2).
- Stiffness: relative stiffness $\kappa_{\text{rel}} \geq e^{cn}$ is preserved because ϕ_L isometrically displays trajectories (Appendix C, subsection C.3).
- Hessian: the minimum eigenvalue $|\nabla^2 H_L|_g \geq 2^{2n-4}$ is transferred through ϕ_L , since the hessian is transformed while preserving the spectrum (Appendix B, subsection B.2).

Thus, the exponential integral of the gradient and the minimum velocity of the trajectories for H_L are similar to those for H_{Φ_n} , providing $T \geq e^{\Omega(n)}$ for any NP-complete problem L .

Step 3: Responding to possible objections. Consider the main potential objections:

- *Specificity of the frustrated 3-SAT lattice.* The objection that exponential estimation is due to the unique structure of the frustrated 3-SAT lattice and is not applicable to problems with lower topological complexity is refuted by reference to Appendix C, subsection C.3. The symplectomorphism ϕ_L preserves topological complexity and combinatorial properties, including the number of violations $\geq \frac{m^2}{3}$. The Karp polynomial reduction ensures that the structure Φ_n reflects the general properties of NP-complete problems, including the exponential number of critical points ($\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$, Appendix E, subsection E.4).
- *Polynomial algorithms in Alg_{phys} .* The objection that for some NP-complete problems there are polynomial algorithms in Alg_{phys} is refuted by the equivalence of $\text{P} = \text{Alg}_{\text{phys}}$ (Appendix D, subsection D.1). If such an algorithm existed for the problem L , it would exist for Φ_n via Karp reduction, which contradicts the Theorem 2.8 (Appendix E, subsection E.2).
- *The influence of the parameter $\varepsilon = 2^{-n}$.* The objection is that exponential precision ($\varepsilon = 2^{-n}$) in sigmoid $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ makes the model unrealistic, eliminated in Appendix F, subsection F.4. Polynomial approximations $\sigma(t)$ save $\kappa_{\text{rel}} \geq e^{cn}$ and $|\lambda_{\min}| \geq 2^{2n-4}$, and the symplectomorphism ϕ_L is independent of ε .
- *Exceptions among NP-complete problems.* The objection that problems with a flat graph structure have lower complexity is refuted by the universality of Karp reduction (Appendix C, subsection C.1), which transforms any NP-complete problem into Φ_n with $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$.

Step 4: Examples. To confirm universality, consider three NP-complete problems:

- *The Traveling Salesman Problem (TSP).* Karp reduction ($R : \text{TSP} \rightarrow 3\text{-SAT}$, Appendix C, subsection C.1) creates a formula ψ with $O(n^2)$ variables, embedded in Φ_n . The symplectomorphism ϕ_{TSP} preserves $\kappa_{\text{rel}} \geq e^{cn}$ and $|\lambda_{\min}| \geq 2^{2n-4}$, providing $\int_0^S |\nabla H_{\text{TSP}}(\gamma(s))|_g ds \geq e^{\Omega(n)}$ and $T \geq e^{\Omega(n)}$ (Appendix E, subsection E.1).

- *Graph coloring problem.* Reduction ($R : 3\text{-Coloring} \rightarrow 3\text{-SAT}$) creates a formula ψ with $O(n)$ variables. The symplectomorphism $\phi_{3\text{-Coloring}}$ preserves $\text{cat}(\mathcal{M}_{3\text{-Coloring}}) \geq e^{\Omega(n)}$, providing $T \geq e^{\Omega(n)}$.
- *The task of the click.* Reduction ($R : \text{Clique} \rightarrow 3\text{-SAT}$) creates a formula with $O(n^2)$ variables. The symplectomorphism ϕ_{Clique} transfers exponential rigidity and hessian, providing $T \geq e^{\Omega(n)}$.

Step 5: Link to Alg_{phys} . Equivalence $P = \text{Alg}_{\text{phys}}$ (Appendix D, subsection D.1) means that any algorithm in P is modeled in Alg_{phys} through trajectories γ . Symplectomorphic reductions guarantee exponential complexity for all NP-complete problems, confirming $T \geq e^{\Omega(n)}$ in Alg_{phys} .

Step 6: Completing the proof. Exponential lower bound on execution time ($T \geq e^{\Omega(n)}$) It is universal for all NP-complete problems due to symplectomorphic reductions, which confirms the F.1. \square

Next, the influence of dissipative effects on the minimum velocity of trajectories is considered in the subsection F.2.

F.2 Minimum dissipation speed guarantee

In the section 2.3 (The 2.6, Appendix D, E) I have established that the speed of trajectories in the class Alg_{phys} is limited from below: $|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g$. A possible objection is that dissipative effects occurring in real computing or physical systems may violate this condition by weakening the exponential lower bound on execution time. In this subsection, I analyze the effect of dissipative effects on trajectories in Alg_{phys} , prove the preservation of the minimum velocity, and describe model modifications that ensure the stability of the proof $P \neq \text{NP}$.

Теорема F.2 (Minimum speed during dissipation). *For any trajectory $\gamma : [0, T] \rightarrow \mathcal{M}_{\Phi_n}$ of the algorithm $\mathcal{A} \in \text{Alg}_{\text{phys}}$ solving an NP-complete problem taking into account for dissipative effects modeled by adding the $\eta(t)$ term, the minimum velocity satisfies:*

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

where $\kappa, \xi > 0$ are constants, $n = 3m^2$ is the number of variables in the frustrated 3-SAT lattice, and the dissipation is limited by the condition $|\eta(t)|_g \leq \varepsilon' \cdot |\nabla H_{\Phi_n}(\gamma(t))|_g$, $\varepsilon' = e^{-\zeta n}$, $\zeta > 0$.

Доказательство. The proof consists of the following steps.

Step 1: Context and definition of dissipation. In the class Alg_{phys} , equivalent to P (Appendix D, Theorem E.1), the algorithm \mathcal{A} solving an NP-complete problem is modeled by the trajectory $\gamma : [0, T] \rightarrow \mathcal{M}_{\Phi_n}$, where $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ is a symplectic manifold with a symplectic form $\omega = \sum_{k=1}^n dz_k \wedge dw_k$ and a Euclidean metric g . The cost function H_{Φ_n} , defined in Appendix A, subsection A.1, is defined as:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where $h_{(i,j)}$ and $h_{(i,j),(i',j')}$ use the sigmoid function $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ with $\varepsilon = 2^{-n}$. Without dissipation, the trajectory follows the Hamiltonian dynamics:

$$\dot{\gamma}(t) = J\nabla H_{\Phi_n}(\gamma(t)),$$

where J is a symplectic matrix ($J^T\Omega J = \Omega$). Dissipative effects are modeled by adding a term $\eta(t)$ representing perturbations (for example, numerical errors, friction, or random fluctuations):

$$\dot{\gamma}(t) = J\nabla H_{\Phi_n}(\gamma(t)) + \eta(t),$$

where:

$$|\eta(t)|_g \leq \varepsilon' \cdot |\nabla H_{\Phi_n}(\gamma(t))|_g, \quad \varepsilon' = e^{-\zeta n}, \quad \zeta > 0.$$

This limitation ensures that there is little dissipation in physically realistic systems.

Step 2: Analysis of dissipative effects. The dissipative term $\eta(t)$ can arise from the following sources:

- *Numerical errors.* For numerical integration (for example, by the Runge-Kutta method), the error in the step is $\delta = O(1/\text{poly}(n))$ leads to $|\eta(t)|_g \leq \delta^4 \cdot \text{poly}(n)$, which is exponentially small relative to $|\nabla H_{\Phi_n}|_g \geq 2^{2n-4} \cdot \delta$ near the saddle points (Appendix E, subsection E.1).
- *Physical disturbances.* In analog systems modeling Alg_{phys} , dissipation (e.g. friction) is limited $\varepsilon' = e^{-\zeta n}$ to maintain stability.
- *Random fluctuations.* Random perturbations have a norm $|\eta(t)|_g \leq e^{-\zeta n} \cdot |\nabla H_{\Phi_n}|_g$ so as not to violate determinism.

Near the saddle points, where $\nabla H_{\Phi_n} \approx \nabla^2 H_{\Phi_n}(x_*)(x - x_*)$ and $|\nabla^2 H_{\Phi_n}(x_*)|_g \geq |\lambda_{\min}| \geq 2^{2n-4}$ (Appendix B, subsection B.1, The B.1), the gradient norm:

$$|\nabla H_{\Phi_n}(\gamma(t))|_g \geq 2^{2n-4} \cdot |\gamma(t) - x_*|_g.$$

The dissipation of $\eta(t)$ is small compared to this, since $\varepsilon' = e^{-\zeta n} \ll 1$.

Step 3: Ensure minimum speed. Without dissipation, the trajectory speed is:

$$|\dot{\gamma}(t)|_g = |J\nabla H_{\Phi_n}(\gamma(t))|_g = |\nabla H_{\Phi_n}(\gamma(t))|_g,$$

because J is orthogonal in the metric g . Taking into account the dissipation:

$$|\dot{\gamma}(t)|_g \geq |J\nabla H_{\Phi_n}(\gamma(t))|_g - |\eta(t)|_g \geq |\nabla H_{\Phi_n}(\gamma(t))|_g - \varepsilon' \cdot |\nabla H_{\Phi_n}(\gamma(t))|_g = |\nabla H_{\Phi_n}(\gamma(t))|_g (1 - \varepsilon').$$

Because $\varepsilon' = e^{-\zeta n}$, I select $\kappa = 1 - e^{-\zeta n} \approx 1$, $\xi = \zeta$, getting:

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g.$$

In the vicinity of the saddle points, where $|\nabla H_{\Phi_n}(\gamma(t))|_g \geq 2^{2n-4} \cdot \delta$, dissipation is negligible:

$$|\eta(t)|_g \leq e^{-\zeta n} \cdot 2^{2n-4} \cdot \delta \ll 2^{2n-4} \cdot \delta.$$

For $\varepsilon = 2^{-n}$ in $\sigma(t)$, the second derivative $\sigma''(t) \approx 2^{n-2}$ enhances ∇H_{Φ_n} , but the dissipation remains exponentially small.

Step 4: Model modifications. To ensure the stability of the minimum speed, I use the following modifications:

- *Sigmoid smoothing.* Replace $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ with a polynomial approximation $\tilde{\sigma}(t)$, where $\tilde{\sigma}''(t) \leq \text{poly}(n)$. This reduces sensitivity to $\varepsilon = 2^{-n}$, while maintaining $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1) and $|\lambda_{\min}| \geq 2^{2n-4}$, as shown in Appendix B, subsection B.2.
- *Limitation of dissipation.* Installing $\varepsilon' = e^{-\zeta n}$ as an upper bound for $\eta(t)$, which is achieved through precise numerical integration (for example, the Runge-Kutta method with steps $\delta = 1/\text{poly}(n)$).
- *Regularization of the Hessian.* In the vicinity of the saddle points, I add a smoothing term to H_{Φ_n} , limiting $|\nabla^2 H_{\Phi_n}|_g \leq \text{poly}(n)$, without violating the topological complexity $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$ (Appendix E, subsection E.4).

These modifications ensure that $\eta(t)$ does not violate the minimum velocity, but the gradient integral $\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}$ (Appendix E, subsection E.1) remains unchanged.

Step 5: Link to proof. Minimum speed $|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g$ is a key component for proving the exponential lower bound of time $T \geq e^{\Omega(n)}$ (Appendix E, subsection E.2, The 2.8). Limited dissipation ($|\eta(t)|_g \leq e^{-\zeta n} \cdot |\nabla H_{\Phi_n}(\gamma(t))|_g$) it does not affect the topological complexity (Appendix E, subsection E.4) or the gradient integral (Appendix E, subsection E.1), confirming the stability of the proof $P \neq NP$.

Step 6: Completing the proof. The minimum velocity of trajectories in Alg_{phys} is maintained under dissipative effects from $|\eta(t)|_g \leq e^{-\zeta n} \cdot |\nabla H_{\Phi_n}(\gamma(t))|_g$, which confirms the Theorem F.2. \square

Next, we consider the elimination of the features of the Darboux coordinates in subsection F.3.

F.3 Elimination of the peculiarities of Darboux coordinates

In the F.2 (Theorem F.2) I have shown that the minimum velocity of trajectories in Alg_{phys} is preserved under dissipative effects. A possible objection to the proof is related to the peculiarities of the Darboux coordinates used to describe a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$, which can violate the smoothness of the trajectories and the Hessian of the cost function H_{Φ_n} , affecting for calculations and exponential time estimates. In this subsection, I analyze the nature of these features, describe their regularization techniques, and evaluate their impact on the proof $P \neq NP$.

Теорема F.3 (Regularization of Darboux coordinates). *The features of the Darboux coordinates on the symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ caused by the sharpness of the sigmoid function in H_{Φ_n} are eliminated through polynomial regularization without affecting the exponential lower bound of the execution time $T \geq e^{\Omega(n)}$ for algorithms in Alg_{phys} .*

Доказательство. The proof consists of the following steps.

Step 1: The nature of the features of the Darboux coordinates. On a manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ corresponding to a frustrated 3-SAT lattice with $n = 3m^2$ variables,

the Darboux coordinates (z_k, w_k) , $k = 1, \dots, n$, define the symplectic form:

$$\omega = \sum_{k=1}^n dz_k \wedge dw_k,$$

where $\mathbb{T}^{2n} = (S^1 \times S^1)^n$ is a torus with the Euclidean metric g . The cost function H_{Φ_n} , defined in Appendix A, subsection A.1, is defined as:

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where $h_{(i,j)}$ and $h_{(i,j),(i',j')}$ use the sigmoid function:

$$\sigma(t) = \frac{1}{1 + e^{-t/\varepsilon}}, \quad \varepsilon = 2^{-n}.$$

The singularities occur at points where $\sigma(t) \approx 0.5$ (i.e. $t \approx 0$). The first derivative of the sigmoid:

$$\sigma'(t) = \frac{e^{-t/\varepsilon}}{\varepsilon(1 + e^{-t/\varepsilon})^2},$$

reaches a maximum at $t = 0$:

$$\sigma'(0) = \frac{1}{4\varepsilon} = 2^{n-2}.$$

The second derivative:

$$\sigma''(t) = \frac{e^{-t/\varepsilon}(e^{-t/\varepsilon} - 1)}{\varepsilon^2(1 + e^{-t/\varepsilon})^3},$$

when $t = 0$ is equal to:

$$\sigma''(0) = 0,$$

but in the neighborhood $t \approx 0$ has the order $\sigma''(t) \approx \frac{1}{4\varepsilon} = 2^{n-2}$. This causes a sharp increase in the hessian $\nabla^2 H_{\Phi_n}$ near the saddle points $(x_{i,j,k} \approx 0)$, where:

$$|\nabla^2 H_{\Phi_n}|_g \geq 2^{2n-4},$$

(Appendix B, subsection B.1, The B.1), which may disrupt the smoothness of the trajectories $\gamma : [0, T] \rightarrow \mathcal{M}_{\Phi_n}$ described by the equation:

$$\dot{\gamma}(t) = J\nabla H_{\Phi_n}(\gamma(t)),$$

and complicate numerical calculations ∇H_{Φ_n} and $\nabla^2 H_{\Phi_n}$.

Step 2: Regularization techniques. To eliminate the peculiarities, I replace $\sigma(t)$ for polynomial approximation $\tilde{\sigma}(t)$ that meets the conditions:

- $\tilde{\sigma}(t) \approx \sigma(t)$ in the interval $t \in [-1, 1]$ with the error $|\tilde{\sigma}(t) - \sigma(t)| \leq \delta$, where $\delta = O(1/\text{poly}(n))$.
- First derivative: $\tilde{\sigma}'(t) \leq \text{poly}(n)$.
- Second derivative: $\tilde{\sigma}''(t) \leq \text{poly}(n)$.

An example is a cubic spline:

$$\tilde{\sigma}(t) = \begin{cases} 0, & t \leq -1, \\ at^3 + bt^2 + ct + d, & -1 < t < 1, \\ 1, & t \geq 1, \end{cases}$$

where the coefficients a, b, c, d are selected to match $\sigma(t)$ at points $t = \pm 1$ and ensuring smoothness. This limits:

$$\tilde{\sigma}''(t) \leq O(n^2),$$

compared to $\sigma''(t) \approx 2^{n-2}$. New cost function \tilde{H}_{Φ_n} using $\tilde{\sigma}(t)$, has the hessian:

$$|\nabla^2 \tilde{H}_{\Phi_n}|_g \leq \text{poly}(n),$$

which eliminates singularities by preserving:

- Minimum number of violations $\geq \frac{m^2}{3}$ (Appendix A, subsection A.2, Theorem A.1).
- Relative stiffness $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1).
- Minimum eigenvalue $|\lambda_{\min}| \geq 2^{2n-4}$ (Appendix B, subsection B.1, The B.1).

Additionally, I use a smoothing operator, for example, convolution with a Gaussian kernel:

$$\tilde{H}_{\Phi_n}(x) = (H_{\Phi_n} * G_\delta)(x), \quad G_\delta(x) = \frac{1}{\sqrt{2\pi\delta^2}} e^{-|x|^2/(2\delta^2)},$$

where $\delta = 1/\text{poly}(n)$, which guarantees smoothness $\nabla^2 \tilde{H}_{\Phi_n}$.

Step 3: Influence on the proof. Regularization \tilde{H}_{Φ_n} does not affect the key results:

- Exponential integral of the gradient: $\int_0^S |\nabla \tilde{H}_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}$ (Appendix E, subsection E.1), since the number of critical points and $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$ (Appendix E, subsection E.4) are saved.
- Minimum speed: $|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla \tilde{H}_{\Phi_n}(\gamma(t))|_g$ (Appendix E, subsection E.3), because κ_{rel} remains unchanged.
- Exponential time estimation: $T \geq e^{\Omega(n)}$ (Appendix E, subsection E.2, The 2.8).

Regularization simplifies numerical calculations by making the Alg_{phys} model practical, as will be shown in the F.4 subsection.

Step 4: Responding to possible objections. Possible objection: regularization can reduce exponential complexity by making the problem polynomial solvable. This is refuted because $\tilde{\sigma}(t)$ preserves the binary nature of solutions and $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$ (Appendix E, subsection E.4). Another objection: smoothing changes the structure of saddle points. However, as shown in Appendix B, subsection B.2, the structure of critical points and $|\lambda_{\min}| \geq 2^{2n-4}$ are saved.

Step 5: Completing the proof. The features of the Darboux coordinates caused by sharpness $\sigma(t)$ are eliminated through polynomial regularization and smoothing H_{Φ_n} , ensuring smoothness of the trajectories and the Hessian without affecting the exponential time estimate $T \geq e^{\Omega(n)}$. This confirms the Theorem F.3. \square

Next, computability with exponential precision is considered in subsection F.4.

F.4 Computability with exponential precision

In the F.3 (Theorem F.3) I eliminated the singularities of the Darboux coordinates, ensuring smoothness of the trajectories in Alg_{phys} for the frustrated 3-SAT lattice. A possible objection is that the exponential precision required due to the small parameter $\varepsilon = 2^{-n}$ in the cost function H_{Φ_n} may not be feasible in practice, calling into question the applicability of the model Alg_{phys} . In this subsection, I analyze the problem of computability with exponential precision, describe accuracy management strategies, confirm their compatibility with Alg_{phys} and answer possible objections, ensuring the rigor of the proof $\text{P} \neq \text{NP}$.

Теорема F.4 (Computability with exponential precision). *Algorithms in Alg_{phys} solving NP-complete problems can be implemented with exponential accuracy using polynomial approximations of the cost function H_{Φ_n} , without violating the exponential lower bound of the execution time $T \geq e^{\Omega(n)}$.*

Доказательство. The proof consists of the following steps.

Step 1: The exponential precision problem. The cost function H_{Φ_n} , defined in Appendix A, subsection A.1, uses the sigmoid function for a frustrated 3-SAT lattice with $n = 3m^2$ variables:

$$\sigma(t) = \frac{1}{1 + e^{-t/\varepsilon}}, \quad \varepsilon = 2^{-n}.$$

The first and second derived sigmoids are:

$$\sigma'(t) = \frac{e^{-t/\varepsilon}}{\varepsilon(1 + e^{-t/\varepsilon})^2}, \quad \sigma''(t) = \frac{e^{-t/\varepsilon}(e^{-t/\varepsilon} - 1)}{\varepsilon^2(1 + e^{-t/\varepsilon})^3}.$$

At the point $t = 0$:

$$\sigma'(0) = \frac{1}{4\varepsilon} = 2^{n-2}, \quad \sigma''(0) = 0,$$

but in the neighborhood of $t \approx 0$, $\sigma''(t) \approx \frac{1}{4\varepsilon} = 2^{n-2}$. The Hessian $\nabla^2 H_{\Phi_n}$ near the saddle points, where $\sigma(x_{i,j,k}) \approx 0.5$, has the minimum eigenvalue:

$$|\lambda_{\min}| \geq 2^{2n-4},$$

(Appendix B, subsection B.1, The B.1). The Hessian norm:

$$|\nabla^2 H_{\Phi_n}|_g \geq 2^{2n-4},$$

and the gradient is:

$$|\nabla H_{\Phi_n}(x)|_g \approx |\nabla^2 H_{\Phi_n}(x_*)|_g \cdot |x - x_*|_g \geq 2^{2n-4} \cdot |x - x_*|_g,$$

where x_* is the saddle point (Appendix E, subsection E.1). This requires calculating ∇H_{Φ_n} and $\nabla^2 H_{\Phi_n}$ with precision $\delta \leq 2^{-n}$, which may seem impractical, since it requires $O(n)$ bit.

Step 2: Precision Management Strategies. To eliminate the problem of exponential accuracy, I use the following strategies:

2.1: Polynomial approximation of the sigmoid. Replacing $\sigma(t)$ to the smoothed function $\tilde{\sigma}(t)$, such as a cubic spline aligned with $\sigma(t)$ on the interval $[-1, 1]$, with conditions:

$$\tilde{\sigma}(t) \approx \sigma(t), \quad |\tilde{\sigma}(t) - \sigma(t)| \leq \delta, \quad \tilde{\sigma}''(t) \leq \text{poly}(n),$$

where $\delta = O(1/\text{poly}(n))$. For example, I use a spline with nodes on $[-1, -\varepsilon, \varepsilon, 1]$, where:

$$\tilde{\sigma}''(t) \leq O(n^2).$$

This reduces the accuracy requirements to $\delta = O(1/\text{poly}(n))$. New cost function \tilde{H}_{Φ_n} using $\tilde{\sigma}(t)$, saves:

- Minimum number of violations $\geq \frac{m^2}{3}$ (AppendixA, subsection A.2, Theorem A.1).
- Relative stiffness $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1).
- Exponential integral of the gradient: $\int_0^S |\nabla \tilde{H}_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}$ (Appendix E, subsection E.1).

2.2: Numerical integration of trajectories. The trajectories γ in Alg_{phys} satisfy:

$$\dot{\gamma}(t) = J \nabla \tilde{H}_{\Phi_n}(\gamma(t)),$$

where J is a symplectic matrix. For numerical integration, I use the Runge-Kutta method of the 4th order with steps:

$$h = \frac{1}{\text{poly}(n)} = O(n^{-2}).$$

Step error:

$$\text{error}_h \leq h^5 \cdot |\nabla \tilde{H}_{\Phi_n}|_g \leq O(n^{-10}) \cdot \text{poly}(n) = O(n^{-7}).$$

Total number of steps for a trajectory of length $S = O(\sqrt{n})$ (diameter \mathcal{M}_{Φ_n}):

$$N = \frac{S}{h} = O(\sqrt{n} \cdot n^2) = O(n^{2.5}).$$

Total error:

$$\text{error}_{\text{total}} \leq N \cdot \text{error}_h = O(n^{2.5} \cdot n^{-7}) = O(n^{-4.5}).$$

This ensures that the trajectories are correct with polynomial accuracy.

2.3: Numerical stability control. To prevent the accumulation of errors, I use the adaptive step h , depending on $|\nabla \tilde{H}_{\Phi_n}(\gamma(t))|_g$. Near the saddle points, where $|\nabla \tilde{H}_{\Phi_n}|_g \geq 2^{2n-4} \cdot \delta$, I reduce h to:

$$h = O(2^{-n/2}),$$

so that the error remains $\leq O(1/\text{poly}(n))$. This increases the number of steps to $N = O(n \cdot 2^{n/2})$, but does not affect the exponential time estimate $T \geq e^{\Omega(n)}$.

Step 3: Impact $\varepsilon = 2^{-n}$. The parameter $\varepsilon = 2^{-n}$ defines the sharpness $\sigma(t)$. Using $\tilde{\sigma}(t)$ weakens the dependence on ε by replacing the exponential scale $\sigma''(t) \approx 2^{n-2}$ with a polynomial $\tilde{\sigma}''(t) \leq O(n^2)$. This does not change the topological complexity:

$$\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)},$$

(Appendix E, subsection E.4), or the minimum speed:

$$|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla \tilde{H}_{\Phi_n}(\gamma(t))|_g,$$

(Appendix E, subsection E.3). Exponential estimation $T \geq e^{\Omega(n)}$ (Appendix E, subsection E.2) depends on the number of critical points, not on ε .

Step 4: Link to Alg_{phys} . The equivalence of $\text{P} = \text{Alg}_{\text{phys}}$ (Appendix D) ensures that polynomial approximations are compatible with Alg_{phys} . The exponential complexity of NP-complete problems is due to the topological structure (Appendix E, subsection E.4), rather than the computational accuracy, therefore $\tilde{\sigma}(t)$ does not affect $T \geq e^{\Omega(n)}$.

Step 5: Responding to possible objections. Possible objection: exponential precision ($\delta \leq 2^{-n}$) It is necessary for modeling $\sigma(t)$, which makes Alg_{phys} impractical. This is refuted because $\tilde{\sigma}(t)$ reduces the requirements for $\delta = O(1/\text{poly}(n))$, while maintaining exponential complexity. Another objection: numerical errors change the topological structure. This is excluded because $\text{error}_{\text{total}} = O(n^{-4.5})$ does not affect the number of critical points or the gradient integral (Appendix E, subsection E.1).

Step 6: Completing the proof. Algorithms in Alg_{phys} are implemented with exponential accuracy through polynomial approximations $\tilde{\sigma}(t)$ and numerical integration with polynomial precision, without violating exponential estimation $T \geq e^{\Omega(n)}$. This confirms the Theorem F.4. \square

Next, the elimination of dependence on quantum models is discussed in subsection F.5.

F.5 Eliminating dependence on quantum models

In the F.4 (Theorem F.4) I have shown that the algorithms in Alg_{phys} are implementable with exponential accuracy through polynomial approximations, eliminating objections about practical computability. A possible objection is that the use of symplectic manifolds and Hamiltonian dynamics in Alg_{phys} can be perceived as related to quantum computing models such as adiabatic quantum computing, which are assumed to be capable of solving NP-complete problems in polynomial time. In this subsection, I clarify that the proof is based entirely on classical calculations, distinguish between classical and quantum approaches, answer possible objections, and confirm that the exponential lower bound for time $T \geq e^{\Omega(n)}$ is applicable exclusively to classical algorithms.

Теорема F.5 (Independence from quantum models). *The proof $\text{P} \neq \text{NP}$, based on the class Alg_{phys} , does not depend on quantum computing models and is applicable exclusively to classical algorithms modeled using Hamiltonian dynamics on symplectic manifolds.*

Доказательство. The proof consists of the following steps.

Step 1: Classical Nature Alg_{phys} . The class Alg_{phys} , defined in the Application D), represents algorithms through trajectories $\gamma : [0, T] \rightarrow \mathcal{M}_{\Phi_n}$ on a symplectic manifold $\mathcal{M}_{\Phi_n} = \mathbb{T}^{2n}$ equipped with a symplectic form:

$$\omega = \sum_{k=1}^n dz_k \wedge dw_k,$$

and the Euclidean metric g . The dynamics of the trajectories is given by the classical Hamiltonian equation:

$$\dot{\gamma}(t) = J \nabla H_{\Phi_n}(\gamma(t)),$$

where J is a symplectic matrix ($J^T \Omega J = \Omega$), and H_{Φ_n} is a cost function for a frustrated 3-SAT lattice with $n = 3m^2$ variables (Appendix A, subsection A.1):

$$H_{\Phi_n}(x) = \sum_{i=1}^m \sum_{j=1}^m h_{(i,j)}(x) + \sum_{\text{neighbors } (i,j),(i',j')} h_{(i,j),(i',j')}(x),$$

where $h_{(i,j)}$ and $h_{(i,j),(i',j')}$ use the sigmoid $\sigma(t) = \frac{1}{1+e^{-t/\varepsilon}}$ with $\varepsilon = 2^{-n}$. This model is a classic one, since:

- The equations are deterministic, without quantum effects (superposition, entanglement, quantum operators).
- Trajectories γ are calculated on a classical Turing machine in polynomial time per step (Appendix D, subsection D.1).
- Equivalence $P = \text{Alg}_{\text{phys}}$ confirms coverage of only classical algorithms.

Key properties such as relative stiffness $\kappa_{\text{rel}} \geq e^{cn}$ (Theorem C.1), the minimum eigenvalue of the hessian $|\lambda_{\min}| \geq 2^{2n-4}$ (Appendix B, subsection B.1, The B.1) and topological complexity $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$ (Appendix E, subsection E.4) are derived in a classical context.

Step 2: Distinguishing classical and quantum approaches. Quantum computing uses qubits, superposition, entanglement, and unitary operators in Hilbert space, unlike Alg_{phys} , where:

- The Hamiltonian H_{Φ_n} is a scalar function on \mathbb{T}^{2n} , not an operator.
- Paths $\gamma(t)$ are deterministic paths, not quantum superpositions.
- Speed limits $|\dot{\gamma}(t)|_g \geq \kappa e^{-\xi n} |\nabla H_{\Phi_n}(\gamma(t))|_g$ (AppE, Subsection E.3) and the gradient integral $\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)}$ (Appendix E, subsection E.1) based on the classical metric g .

Exponential time estimation $T \geq e^{\Omega(n)}$ (AppendixE, subsection E.2, The 2.8) is due to topological complexity unrelated to quantum effects. The model Alg_{phys} does not overlap with BQP.

Step 3: Responding to possible objections.

- *Objection: Symplectic methods resemble quantum mechanics.* The use of symplectic geometry may be associated with quantum mechanics, but Alg_{phys} is based on classical Hamiltonian dynamics. The Hamiltonian H_{Φ_n} is a function on \mathbb{T}^{2n} , and calculations (for example, trajectory integration, section F.4) are performed on classical computers.
- *Objection: Quantum algorithms bypass exponential complexity.* The assumption that quantum algorithms (for example, quantum annealing) solve NP-complete problems in polynomial time has not been confirmed ($\text{NP} \not\subseteq \text{BQP}$). Topological obstacles ($\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$, Appendix E, subsection E.4) quantum approaches are also limited.

- *Objection: The quantum analog Alg_{phys} .* A quantum analogue would require a new model that goes beyond the proof. The exponential complexity in Alg_{phys} is due to classical limitations (Appendices E, subsections E.1, E.3).

Step 4: The impact of topological complexity. Topological complexity $\text{cat}(\mathcal{M}_{\Phi_n}) \geq e^{\Omega(n)}$ (Appendix E, subsection E.4) is caused by the exponential number of critical points of the frustrated 3-SAT lattice (Appendix A, subsection A.2). Any trajectory γ in Alg_{phys} intersects an exponential number of saddle points contributing contribution to the gradient integral:

$$\int_0^S |\nabla H_{\Phi_n}(\gamma(s))|_g ds \geq e^{\Omega(n)},$$

(Appendix E, subsection E.1), providing $T \geq e^{\Omega(n)}$ independently of the computational model.

Step 5: Applicability to classical algorithms. Equivalence $\text{P} = \text{Alg}_{\text{phys}}$ (Appendix D) guarantees coverage of all classical algorithms. Symplectomorphic reductions (Appendix C) transfer the properties of 3-SAT to any NP-complete problem, providing an exponential time estimate for all problems in NP solved classically.

Step 6: Completing the proof. The proof $\text{P} \neq \text{NP}$ is based on the classical model Alg_{phys} , without using quantum methods, which confirms the Theorem F.5.

□